DEPARTAMENTO DE CIENCIAS DE LA COMPUTACIÓN
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SAN FRANCISCO

Fair Exchange of Short Signatures without Trusted Third Party

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Digital Goods Economy

Enforcing Secure Transactions through a Trusted Third Party (TTP)

PayPal™

Google™ Checkout

amazon.com®

Problems with TTP

Anonymous Claims To Have Hacked 28,000 PayPal Passwords For Guy Fawkes Day

The Huffington Post | By Cavan Sieczkowski

Posted: 11/05/2012 11:15 am EST Updated: 11/05/2012 1:01 pm EST

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Problems with TTP

PayPal

Privacy Policy

Last Update: Jul 13, 2010

Jump to section:

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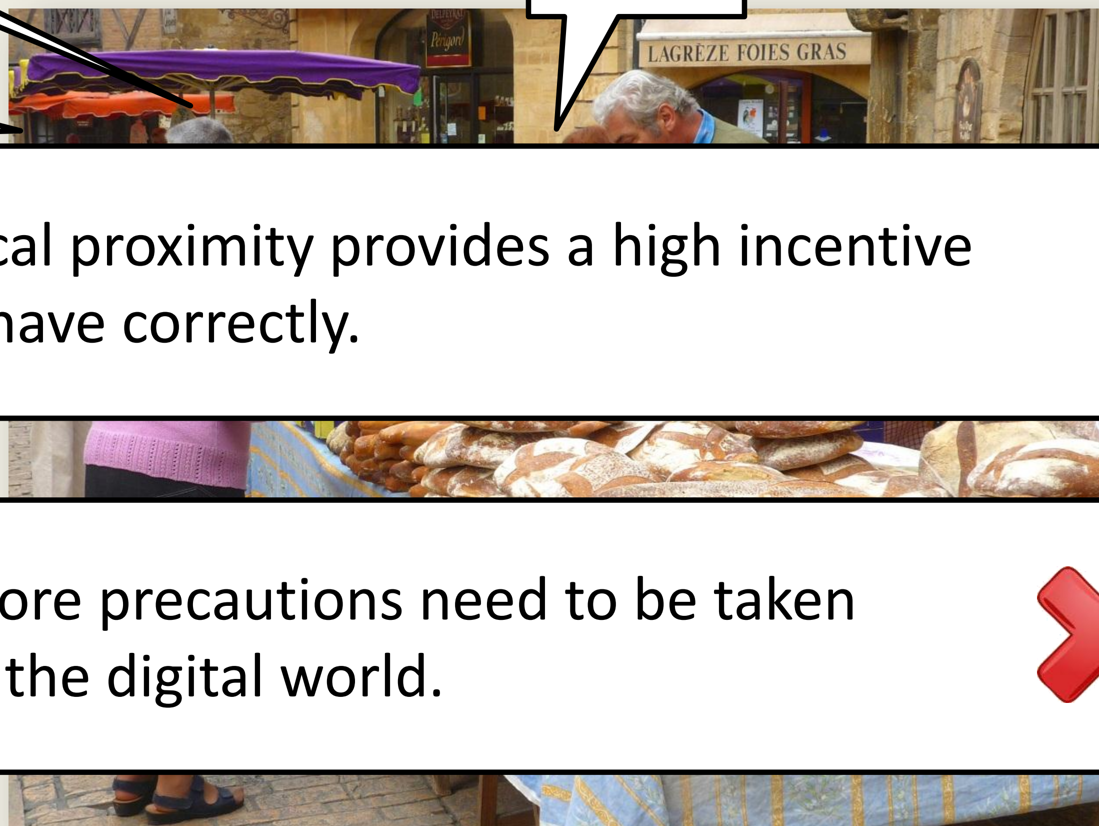
How we collect information about you

Fair Exchange in the Physical World is “easy”

Witness

Witness

Witness



Buyer

Physical proximity provides a high incentive to behave correctly.

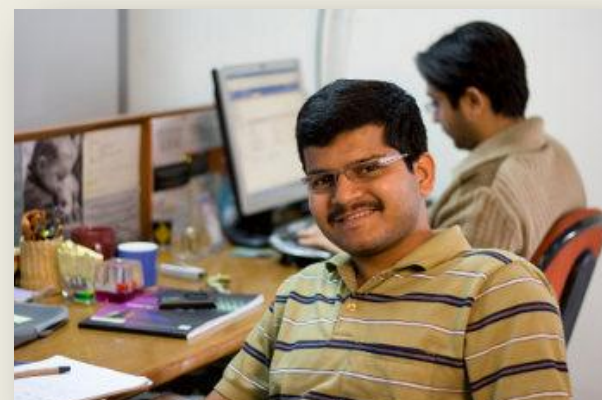
More precautions need to be taken in the digital world.

Modeling Transactions with Digital Signatures

The problem: Who starts first?
Impossibility Result **[Cleve86]**

Software License

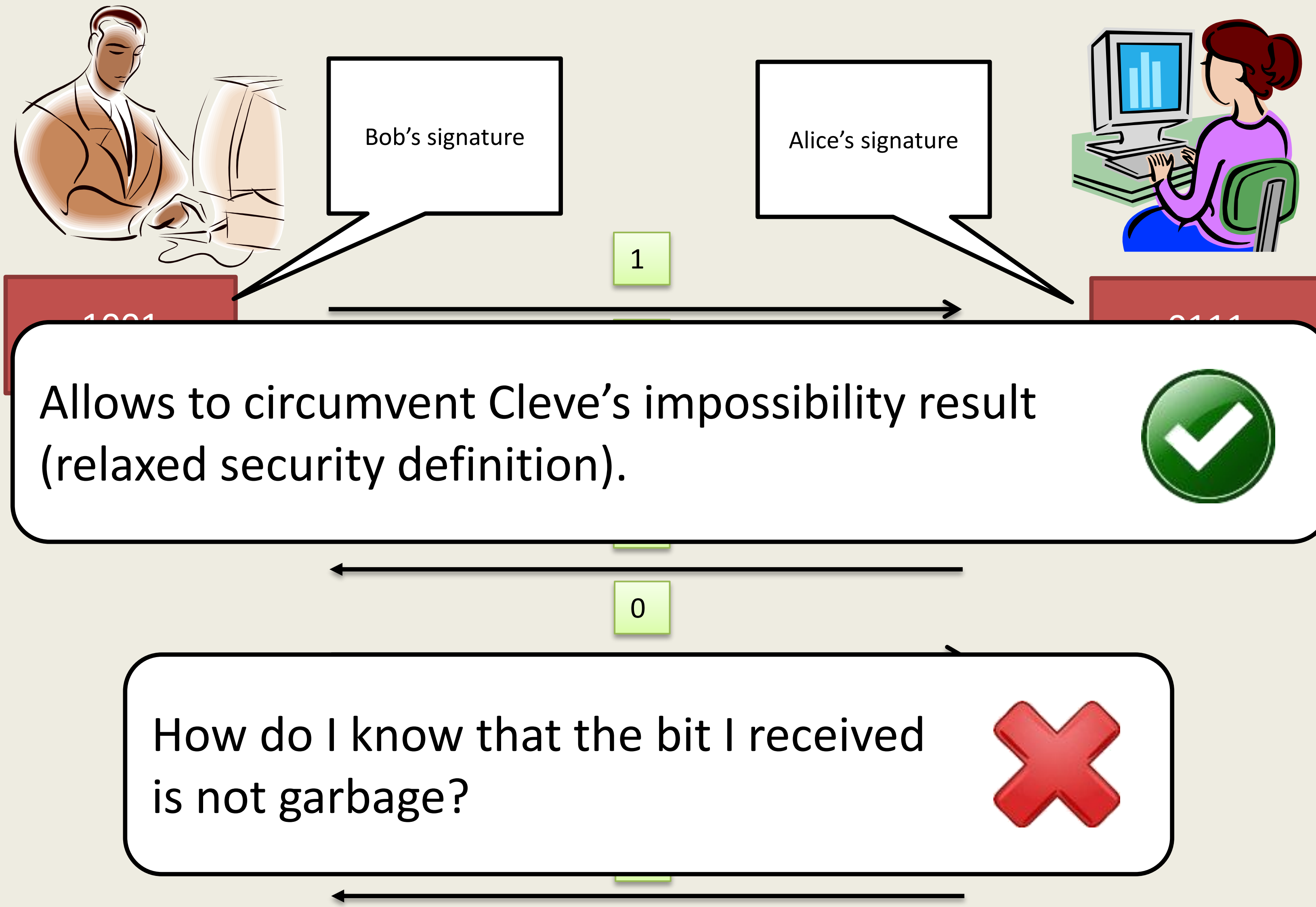
Buyer



Seller

Digital Check

Gradual Release of a Secret



Allows to circumvent Cleve's impossibility result (relaxed security definition).

How do I know that the bit I received is not garbage?

Our Construction

- Fair Exchange of Digital Signatures
- Boneh-Boyen [BB04] Short Signatures
- No TTP
- Practical

Contributions

- Formal definition of *Partial Fairness*

- Efficiency

	κ: Security Parameter	$\boldsymbol{\kappa = 160}$
# Rounds	$\kappa + 1$	161
Communication	$16\kappa^2 + 12\kappa$ bits	≈ 52 kB
# Crypto operations per participant	$\approx 30\kappa$	≈ 4800

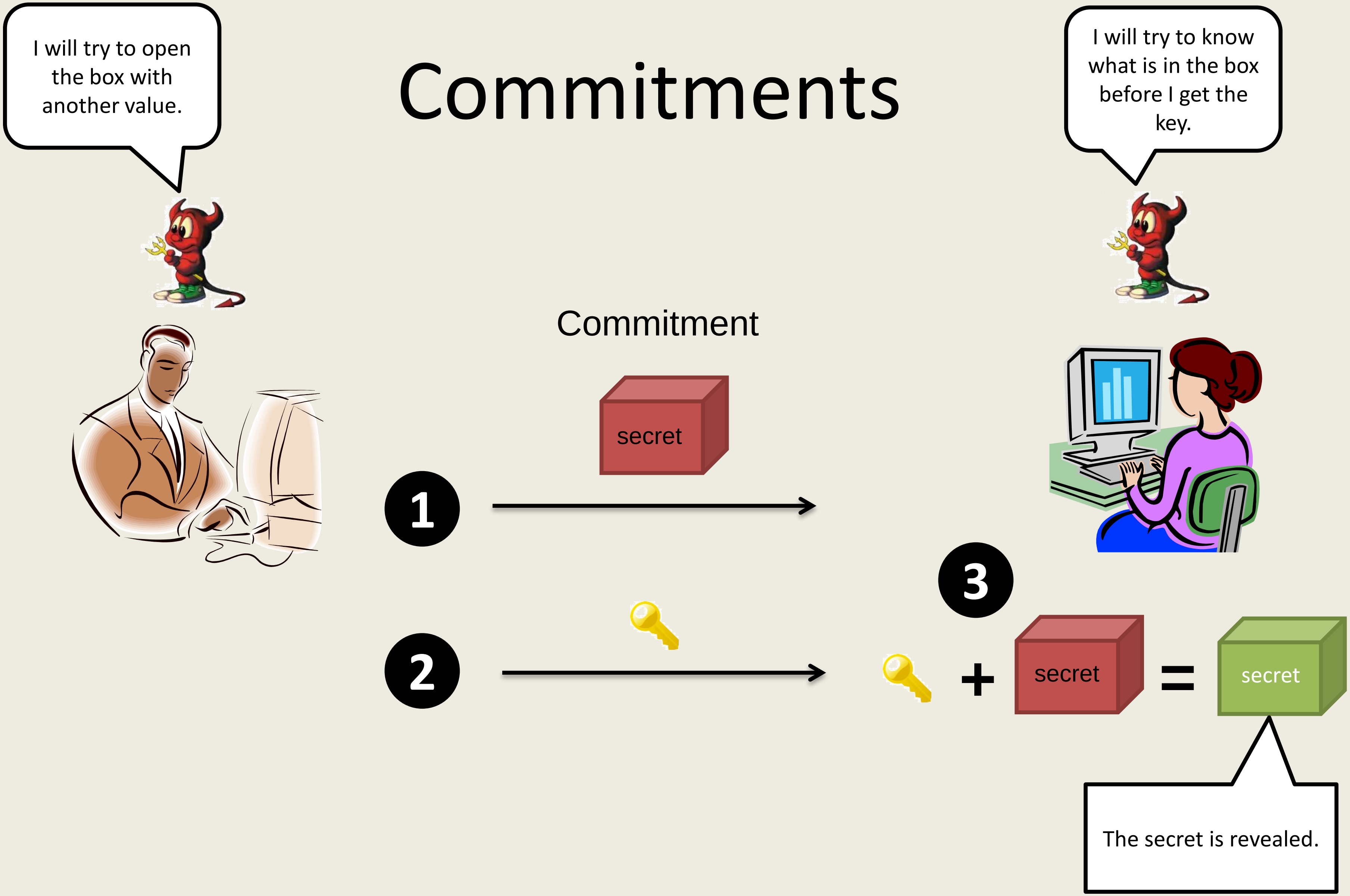
- First protocol for Boneh-Boyen signatures

Contributions

- NIZK argument to prove that a commitment encodes a **bit vector**.

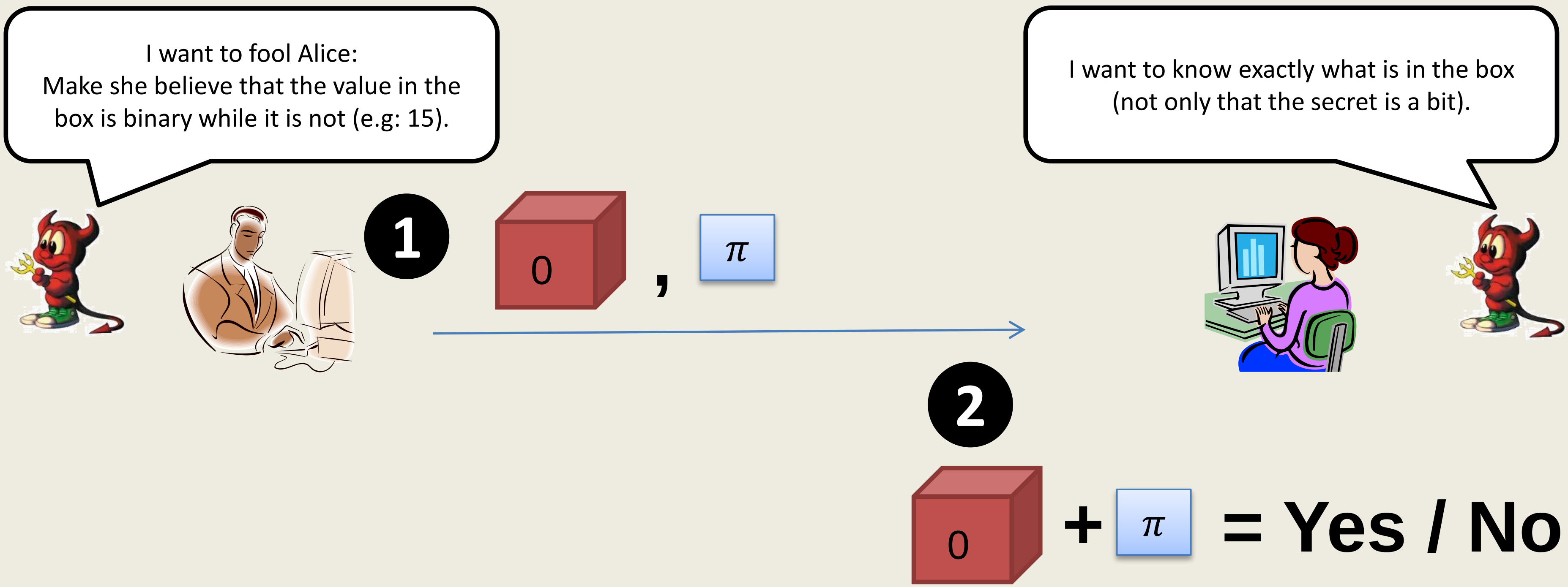
- NIZK argument to prove a commitment to a **bit vector** is the **binary expansion of the discrete logarithm** θ of $D = g^{\theta}$.

Commitments



Non-Interactive
Zero-Knowledge Proofs

Prove something about the secret in the box
without opening the box.



Abstract Protocol

Phase	$\mathcal{P}_A(\mathsf{CRS}, m_A, m_B)$		$\mathcal{P}_B(\mathsf{CRS}, m_A, m_B)$
KeyGen	$(sk_A, pk_A) \leftarrow \mathsf{FEKeyGen}(1^\kappa)$		
	pk_A	$\longrightarrow$	
			$(sk_B, pk_B) \leftarrow \mathsf{FEKeyGen}(1^\kappa)$
		$\longleftarrow$	pk_B
Encrypt Signature	$(\theta_A, \vec{r}_A, \gamma_A) \leftarrow \mathsf{EncSigGen}(\mathsf{CRS}, sk_A, m_A)$		
	γ_A	$\longrightarrow$	
			$(\theta_B, \vec{r}_B, \gamma_B) \leftarrow \mathsf{EncSigGen}(\mathsf{CRS}, sk_B, m_B)$
		$\longleftarrow$	γ_B
Verify Encrypted Signature	$v \leftarrow \mathsf{EncSigCheck}(\mathsf{CRS}, pk_B, m_B, \gamma_B)$		
	if $v = 0$ **then ABORT**		
			$v \leftarrow \mathsf{EncSigCheck}(\mathsf{CRS}, pk_A, m_A, \gamma_A)$
			if $v = 0$ **then ABORT**
Release Bits	**for** $i = 1$ **to** κ:		
	$\mathsf{open}_{A,i} \leftarrow \mathsf{KeyBitProofGen}(\mathsf{CRS}, \vec{r}_A, \theta_A, i)$		
	$\mathsf{open}_{A,i}$	$\longrightarrow$	
			$\mathsf{open}_{B,i} \leftarrow \mathsf{KeyBitProofGen}(\mathsf{CRS}, \vec{r}_B, \theta_B, i)$
		$\longleftarrow$	$\mathsf{open}_{B,i}$
	$v_i \leftarrow \mathsf{KeyBitCheck}(\mathsf{CRS}, \mathsf{open}_{B,i}, i)$		
	if $v_i = 0$ **then ABORT**		
			$v_i \leftarrow \mathsf{KeyBitCheck}(\mathsf{CRS}, \mathsf{open}_{A,i}, i)$
			if $v_i = 0$ **then ABORT**
	end for		
Recover Signature	$\sigma_{m_B} \leftarrow \mathsf{EncSigDecrypt}(\gamma_B, \theta_B)$		
			$\sigma_{m_A} \leftarrow \mathsf{EncSigDecrypt}(\gamma_A, \theta_A)$

Setup

Partial Fairness

$O_{Sign}(sk_B, \cdot)$

Not queried to

m_A, m_B, pk_A

(sk_B, pk_B)

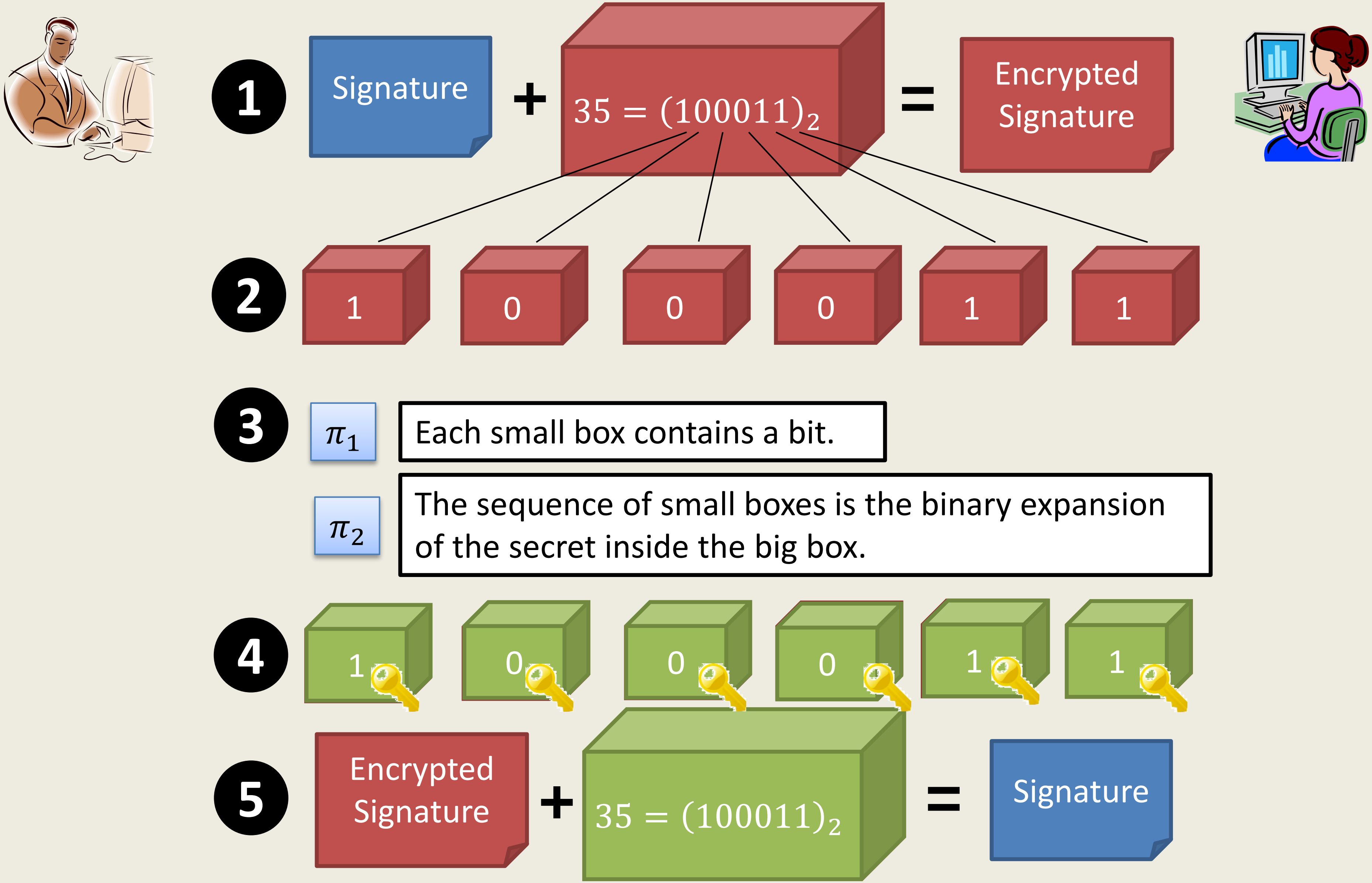
$$\frac{\Pr[\,\mathsf{SVf}(pk_B, m_B, \sigma_A) = \mathsf{valid}\,]}{\Pr[\,\mathsf{SVf}(pk_A, m_A, \sigma_B) = \mathsf{valid}\,]} \leq Q(\kappa)$$

σ_B on m_A

Bet according to
partially released secret

σ_A on m_B

Protocol


1
Signature
+
$35 = (100011)_2$
=
Encrypted Signature
2
1
0
0
0
1
1
3
π_1
Each small box contains a bit.
π_2
The sequence of small boxes is the binary expansion of the secret inside the big box.
4
1
0
0
0
1
1
5
Encrypted Signature
+
$35 = (100011)_2$
=
Signature

Bilinear maps

- $(p, e, G, G_T, g) \leftarrow BMGen(1^k)$

- $|G| = |G_T| = p$
- $e: G \times G \rightarrow G_T$
- $e(g^a, g^b) = e(g,g)^{ab}$
- $e(g,g)$ generates G_T

Assumptions

- Given $(g, g^s, g^{s^2}, g^{s^3}, \cdots, g^{s^q})$ it's hard to compute
  - $g^{\frac{1}{s}}$ (q- Diffie-Hellman Inversion)
  - $e(g, g,)^{\frac{1}{s}}$ (q-Bilinear Diffie-Hellman Inversion)
  - $(c, g^{\frac{1}{s+c}})$ (q-Strong Diffie-Hellman)
  - $g^{s^{q+i}}$ for $1 \leq i \leq q$
    ($q + i$ Diffie-Hellman Exponent)

Assumptions

- **Proposition:** $q - BDHI \Rightarrow \ q + i \ - DHE$
- Our protocol is secure under
  - $q - SDH$
  - $q - BDHI$

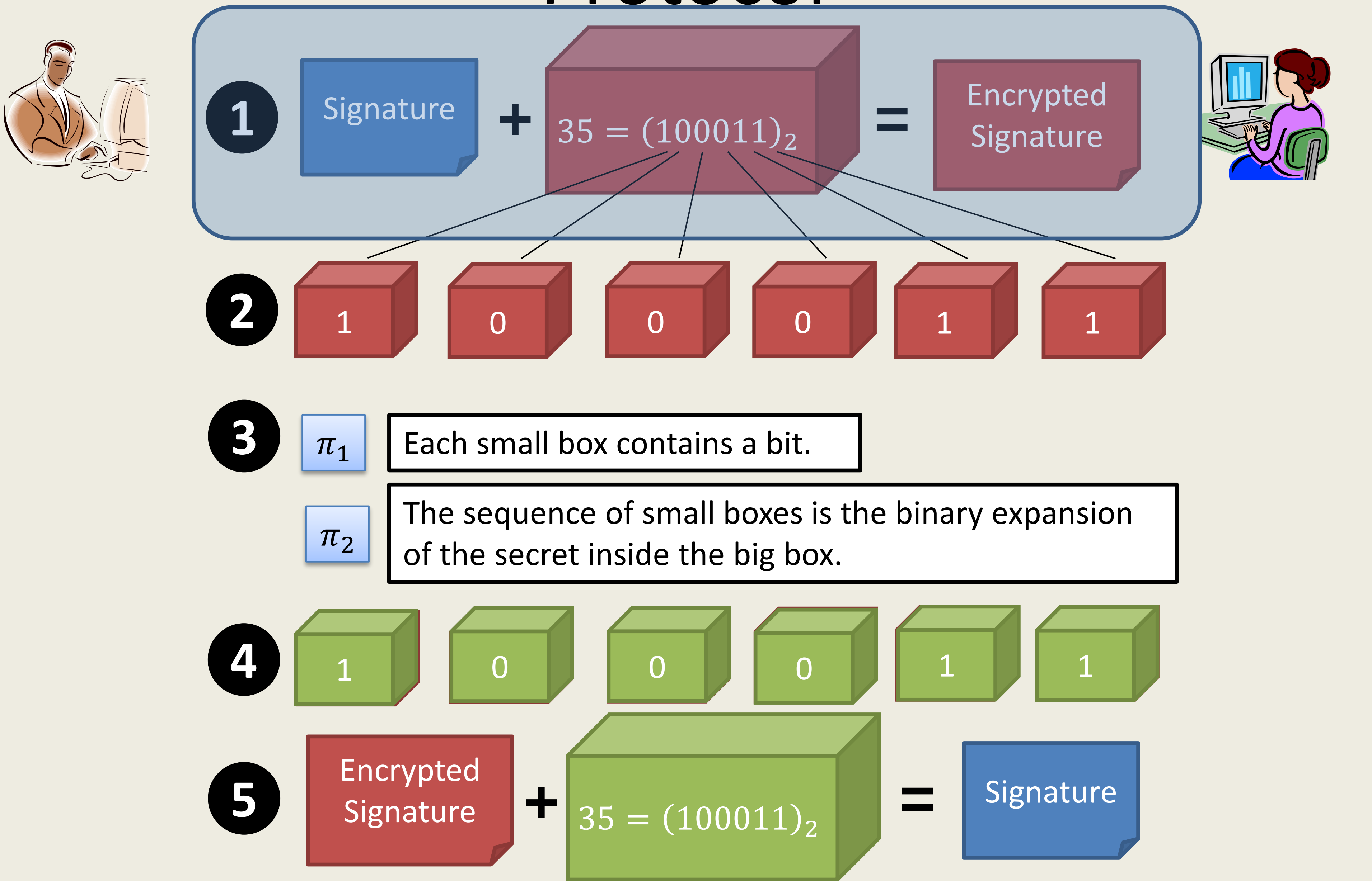
Short Signatures w/o Random Oracle [BB04]

- $\boldsymbol{KeyGen(1^k)}$
  1. $x, y \in Z_p$
  2. $u = g^x, v = g^y$
  3. $pk = (u, v), sk = (x, y)$
  4. return (sk, pk)

- $\boldsymbol{SSign(sk, m)}$
  1. $r \in Z_p$
  2. return $\sigma = (g^{\frac{1}{x+m+yr}}, r) = (\sigma_r, r)$

- $\boldsymbol{SVf(pk, m, \sigma)}$
  1. Check that $\boldsymbol{e(\sigma_r, ug^m v^r)} = e(g^{\frac{1}{x+m+yr}}, g^{x+m+yr}) = \boldsymbol{e(g, g)}$

Protocol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0
0
0
1
1
5
Encrypted Signature
+
35 = (100011)2
=
Signature

The Encrypted Signature

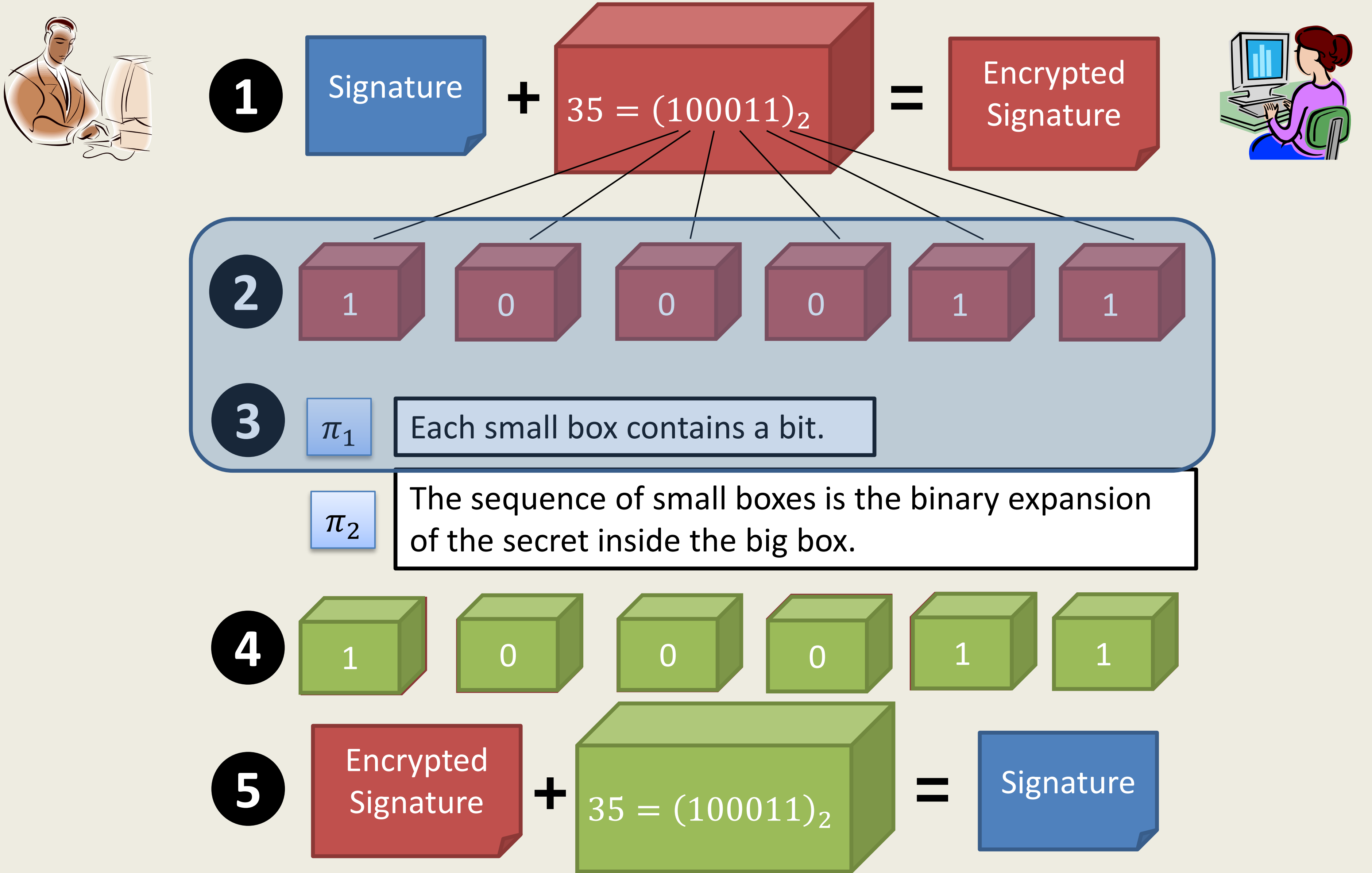
- Computing
  - $\theta \leftarrow Z_p$
  - $D = g^{\theta}$
  - $\boldsymbol{\sigma} = (\boldsymbol{g}^{\frac{\theta}{x+m+yr}}, \boldsymbol{r})$

Secret key / “blinding” factor

Boneh-Boyen signature “blinded” by θ

- Checking
  - Given (D, σ, pk, m) parse σ and pk as
    - $\sigma = (\sigma_\theta, r)$
    - $pk = (g, u = g^x, v = g^y)$
  - $\boldsymbol{e}(\boldsymbol{\sigma_\theta}, \boldsymbol{u}\boldsymbol{g}^{\boldsymbol{m}}\boldsymbol{v}^{\boldsymbol{r}}) = e(g^{\frac{\theta}{x+m+yr}}, g^{x+m+yr}) = \boldsymbol{e}(\boldsymbol{D}, \boldsymbol{g})$

Protocol



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NIZK argument 1

- $CRS = (g, g^{s}, g^{s^2}, g^{s^3}, \cdots, g^{s^q}) = (g_0, g_1, g_2, g_3, \ldots, g_q\ )$
- **Statement**

  Let $C\ = (C_1, C_2, \ldots, C_q)$

  The prover knows $(r_i, b_i) \in\ (Z_p \times\ \{0,1\})$ such that $\boldsymbol{C_i = g^{r_i} g_i^{b_i}}$
- **Argument**
  - $A_i\ =\ g_{q-i}^{r_i}\, g_q^{b_i}$
  - B_i such that $e(A_i, C_i g_i^{-1}) = e(B_i, g)$
  - Return (A_i, B_i) for each $i \in [1..q]$
- **Verification**
  - $e(A_i, g) = e(C_i, g_{q-i})$
  - $e(A_i, C_i g_i^{-1}) = e(B_i, g)$

Shift C_i by $q - i$ positions to the right.

Force the product $\boldsymbol{b_i(b_i - 1)}$ to be computed in the exponent.

NIZK argument 1

- **Theorem:**

  The argument is perfectly complete, computationally sound under the $q+i$ - DHE assumption and perfectly zero-knowledge.

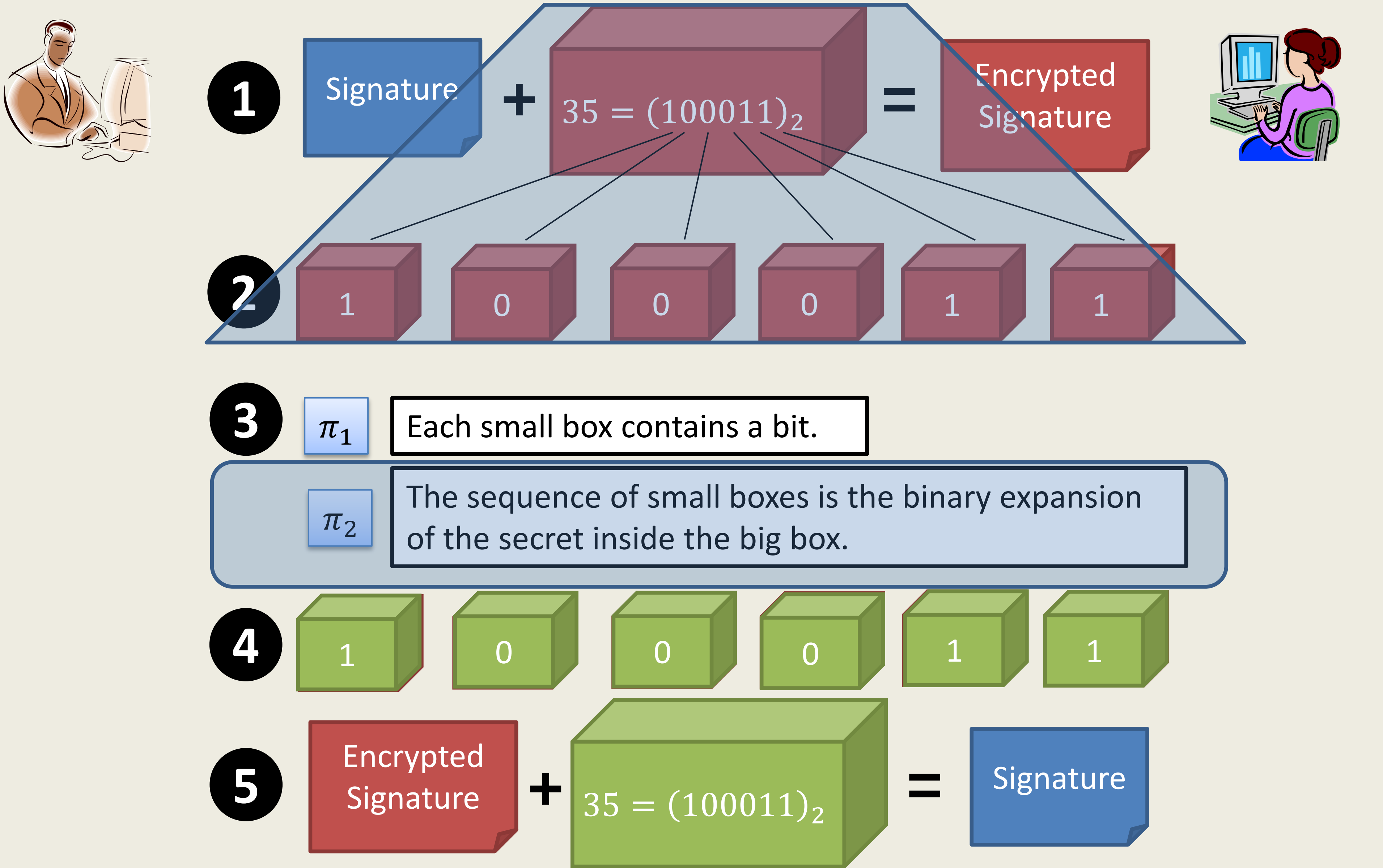
Proof (sketch).

$$e(A_i, C_i g_i^{-1}) = e(g_{q-i}^{r_i} g_q^{b_i}, g^{r_i} g_i^{b_i - 1})$$

$$= e\Big(\underbrace{g_{q-i}^{r_i^2} g_q^{r_i(2b_i-1)}}_{B_i} \boldsymbol{g_{q+i}^{b_i(b_i-1)}}, g\Big) = e(B_i, g)$$

If $b_i \notin \{0,1\}$, the adversary breaks the $q+i-\mathrm{DHE}$ assumption.

Protocol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0
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0
1
1
5
Encrypted Signature
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35 = (100011)2
=
Signature

NIZK argument 2

- $CRS = \left(g, g^{s}, g^{s^2}, g^{s^3}, \cdots, g^{s^q}\right) = \left(g_0, g_1, g_2, g_3, \dots, g_q\right)$

- We set $q = \kappa$ (security parameter)

- **Statement**
  - The prover knows $(r_i, b_i) \in (Z_p \times \{0,1\})$ and θ such that $C_i = g^{r_i} g_i^{b_i}$, $D = g^{\theta}$ and

$$\theta = \sum_{i=1}^{\kappa} b_i 2^{i-1}$$

NIZK argument 2

- **Verification:** Input (C …
  - Parse $\pi = (r', U, V)$
  - Check that $e(\frac{\prod_{i=1}^{k} C_i}{g^{r'}}, g) = e(U, g_1)$
  - Check that $e(\frac{U}{D}, g) = e(V, g_1 g^{-2})$

$$\prod_{i=1}^{k} C_i = \prod_{i=1}^{k} g^{r_i} g_i^{b_i} \Leftrightarrow [r', b_1, b_2, \ldots, b_\kappa]$$

$$U = (\prod_{i=1}^{k} g_i^{b_i})^{1/s} = \prod_{i=1}^{k} g_{i-1}^{b_i} \Leftrightarrow [b_1, b_2, \ldots, b_\kappa]$$

$$r' = \sum_i r_i$$

θ

$U \Leftrightarrow P(s)$ (i.e. $U = g^{P(s)}$)

$V \Leftrightarrow W(s)$ s.t. $P(s) - P(2) = W(s)(s-2)$

NIZK argument 2

- **Theorem:**

  The argument is perfectly complete, computationally sound under the $q - SDH$ assumption and perfectly zero-knowledge.

Protocol

1 Signature + $35 = (100011)_2$ = Encrypted Signature

2 1 0 0 0 1 1

3 π_1 Each small box contains a bit.

π_2 The sequence of small boxes is the binary expansion of the secret inside the big box.

4 1 0 0 0 1 1

5 Encrypted Signature + $35 = (100011)_2$ = Signature

Recovering the Signature

- All the bits b_i are revealed
- Compute $\theta = \sum_{i=1}^{\kappa} b_i \, 2^{i-1}$
- We have $\sigma = \left(g^{\frac{\cancel{\theta}}{x+m+yr}}, r\right) = (\sigma_\theta, r)$
- Compute $\boldsymbol{\sigma} = (\sigma_\theta{}^{\mathbf{1}/\boldsymbol{\theta}}, \boldsymbol{r})$

Proofs of Knowledge

- Discrete logarithm θ of
  - $D = g^{\theta}$
- r_i, b_i such that
  - $C_i = g^{r_i} g_i^{b_i}$

Needed in order to simulate the adversary despite it aborts early.

Simultaneous Hardness of Bits for Discrete Logarithm

Holds in the generic group model **[Schnorr98]**

An adversary cannot distinguish between a **random sequence** of $\boldsymbol{\kappa - l}$ bits and the **first** $\boldsymbol{\kappa - l}$ bits **of** $\boldsymbol{\theta}$ given $\boldsymbol{g^{\theta}}$.

$$Adv^{SHDL}(\mathcal{A},\kappa) = \left|\Pr\left[\begin{array}{c}\theta \xleftarrow{R} \mathbb{Z}_p : \\ 1 \leftarrow \mathcal{A}(g^{\theta}, \theta[1..\kappa - l])\end{array}\right] - \Pr\left[\begin{array}{c}\theta, \alpha \xleftarrow{R} \mathbb{Z}_p : \\ 1 \leftarrow \mathcal{A}(g^{\theta}, \alpha[1..\kappa - l])\end{array}\right]\right|$$

$l = \omega(\log \kappa)$

Conclusion

- Fair exchange protocol for short signatures [BB04] without TTP
- Practical
- Two new NIZK arguments

Thank you!

Partial Fairness

- A randomized protocol for signing contracts [EGL85]
- Gradual release of a secret [BCDB87]
- Practically and Provably secure release of a secret and exchange of signatures [Damgard95]
- Resource Fairness and Composability of Cryptographic protocols [GMPY06]

Only contract signing

RSA, Rabin, ElGamal signatures

"Time-line" assumptions, Generic construction

- **Theorem:**

  The protocol is partially fair under the $\kappa - SDH$ and the $\kappa - BDHI$ assumption.

Proof (Sketch)

- Type I
  - Does not forge values but aborts «early»
  - => He has to break the signature scheme

  - **Careful:**
    What happens if A detects he is simulated?
    - The simulator will try to break the SHDL assumption
    - If few bits remain, it does not win, everything is OK!

Proof (Sketch)

- Type II
  - Forge values
  - The simulator can extract all values computed by adversary and break the soundness of the NIZK arguments or binding property of commitment scheme.