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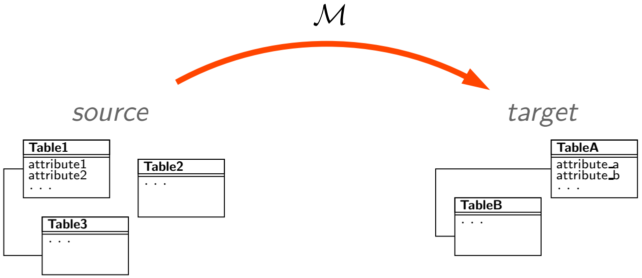

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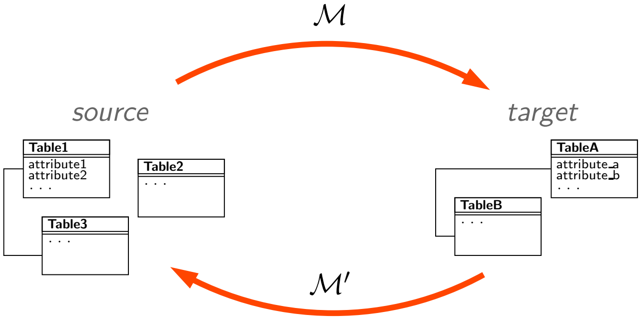

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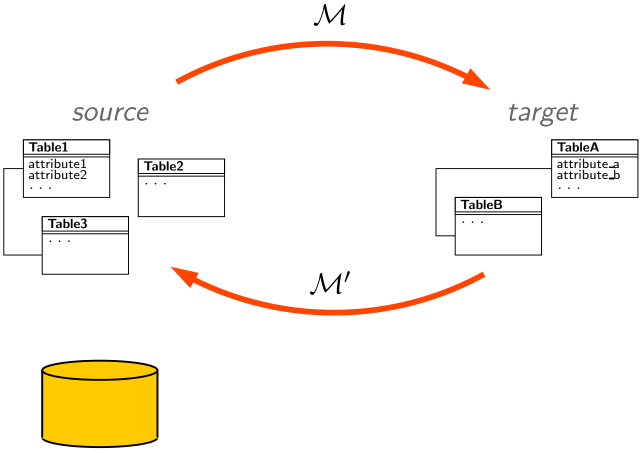

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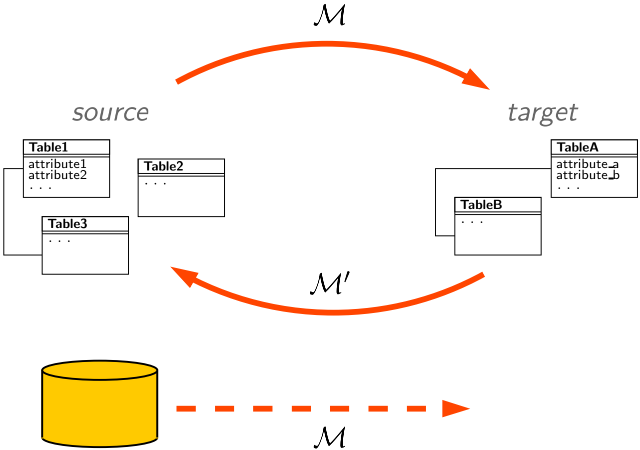

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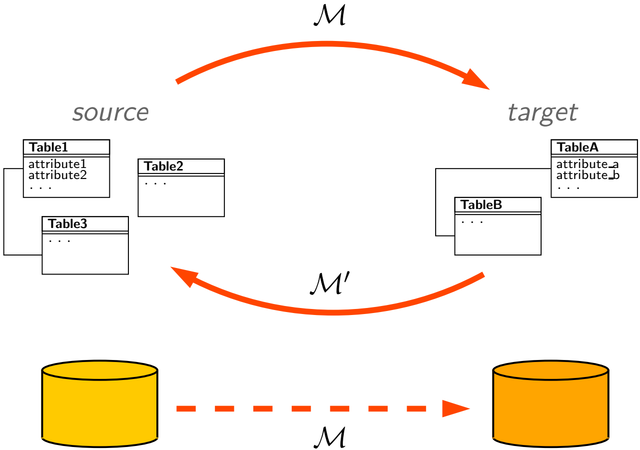

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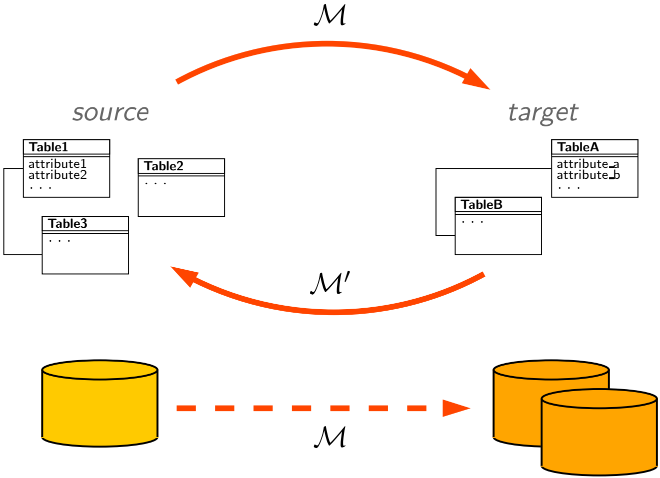

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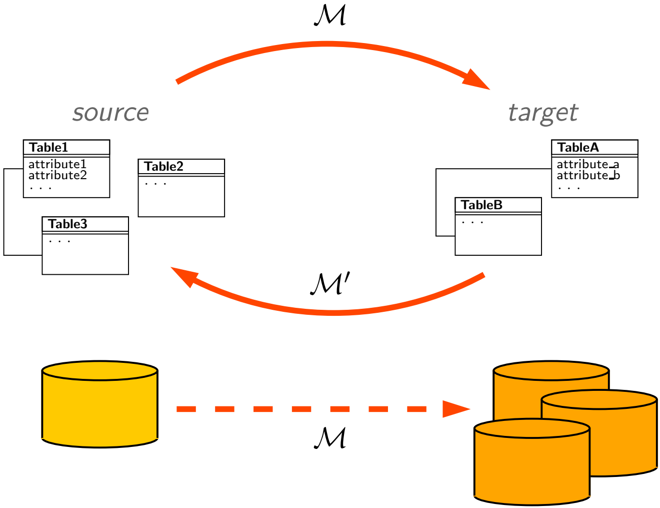

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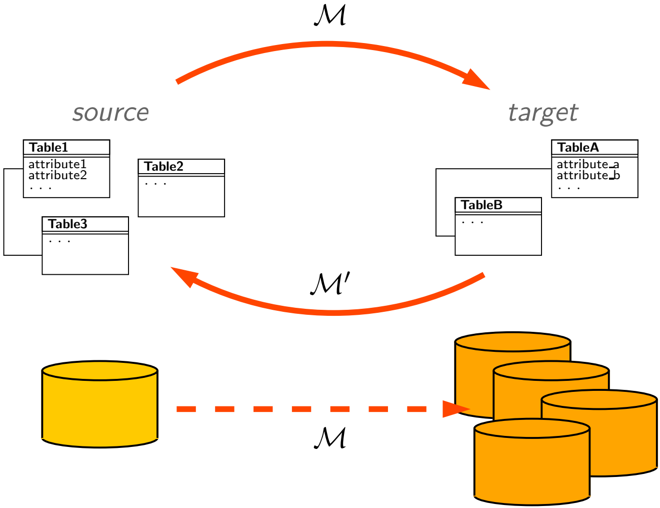

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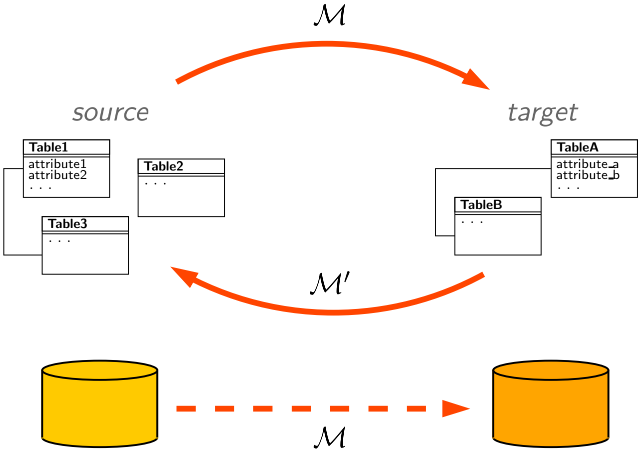

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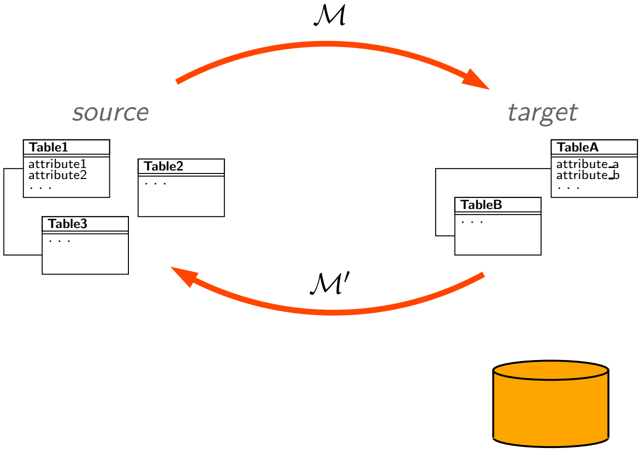

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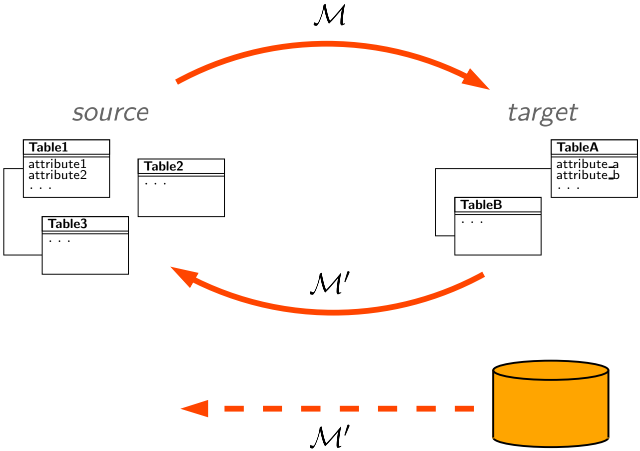

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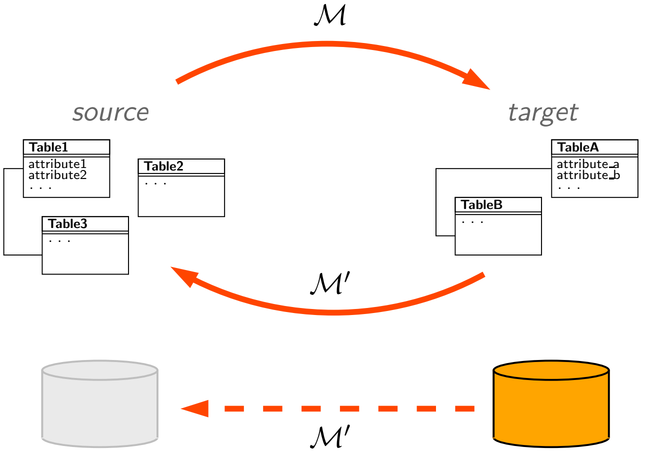

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We propose solutions to the above issues.

Inverting Schema Mappings: Bridging the Gap between Theory and Practice

Marcelo Arenas, Jorge Pérez, Cristian Riveros, Juan Reutter

Computer Science Department, PUC – Chile

Main results

1. A new notion of inverse based on queries
2. A proof that when focusing on **CQ**, inverses of tgds can be expressed in a language that has the same good properties as tgds for data exchange (we provide an algorithm).
3. Algorithm for computing all the previous notions of inverse in polynomial time (drawback: uses an SO language)

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In this talk only 1 and 2

A bit of notation...

A *mapping* $\mathcal{M}$ is a set of pairs (I, J) with

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We say that $\mathcal{M}$ is specified by a set of formulas Σ if

- $(I, J) \in \mathcal{M}$ iff (I, J) satisfies Σ.

Certain answers: tuples present in all the solutions

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$$\begin{aligned} \mathcal{M}: &\quad A(x,y) \rightarrow \exists Z \; T(x,Z) \wedge R(Z,y) \\ I: &\quad \{ \; A(1,1) \; \} \end{aligned}$$

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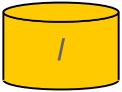
Recovering sound information wrt a query

source *target*



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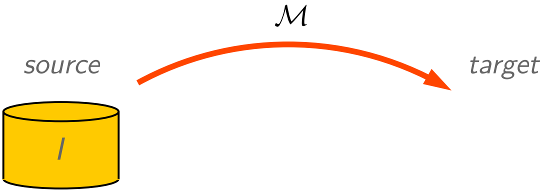

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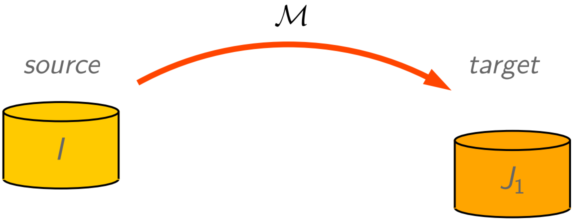

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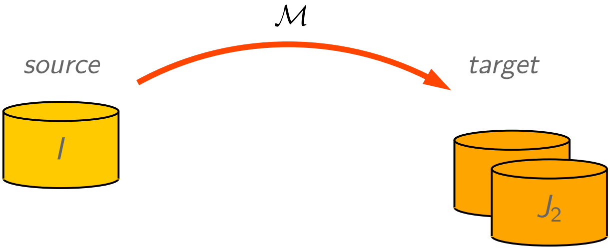

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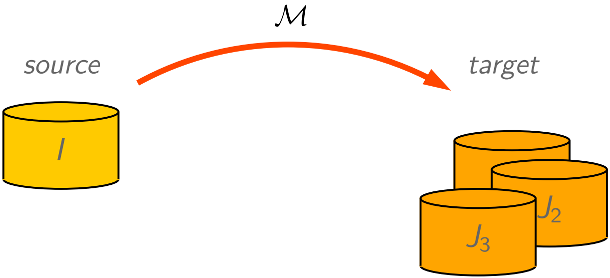

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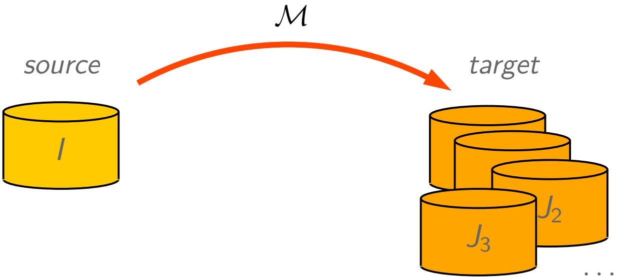

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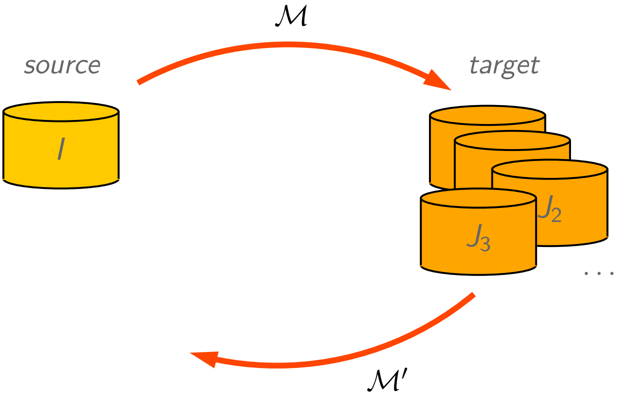

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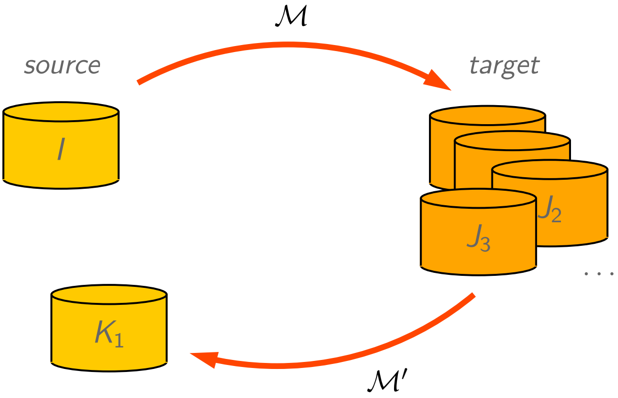

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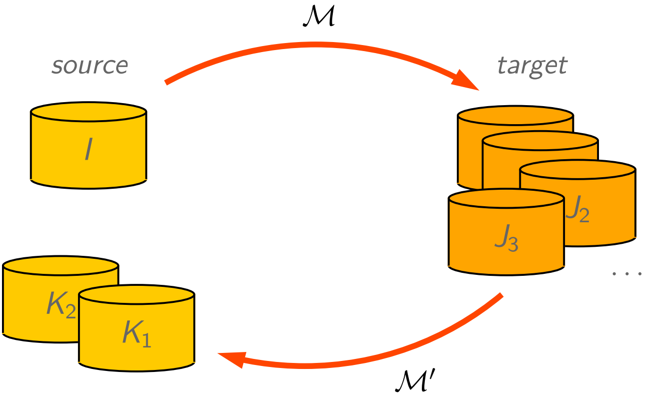

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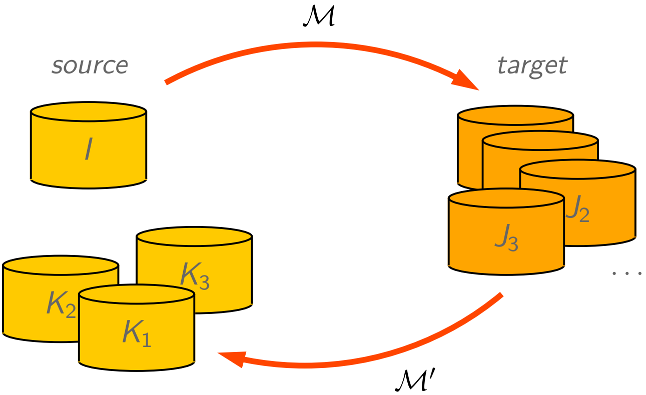

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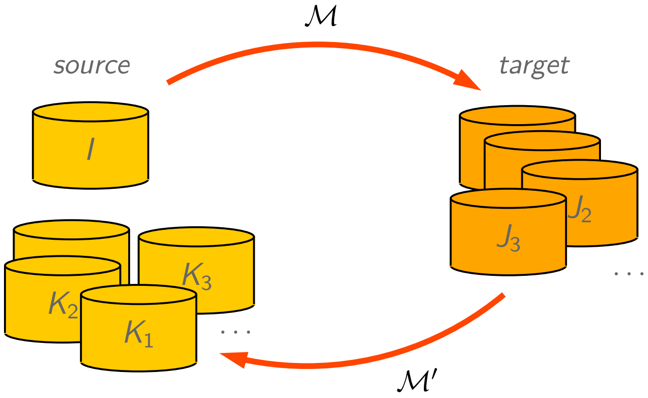

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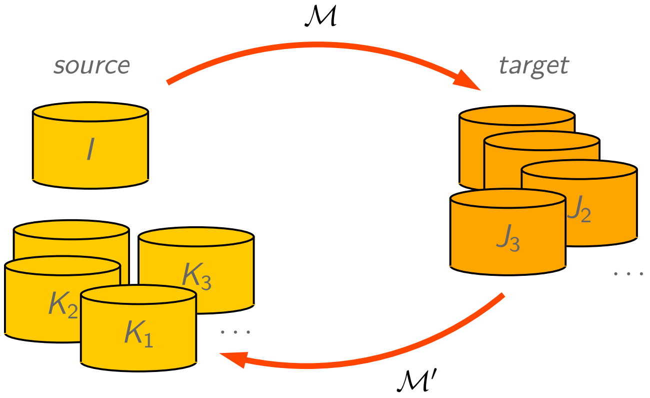

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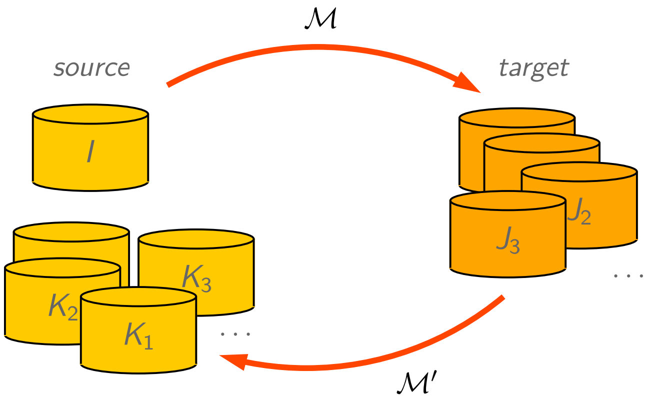

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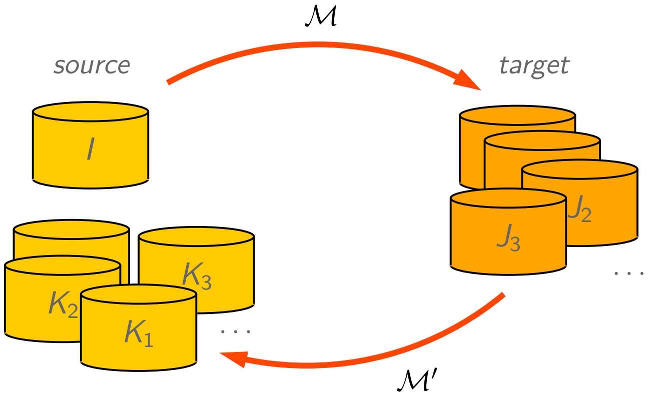
Recovering sound information wrt a query



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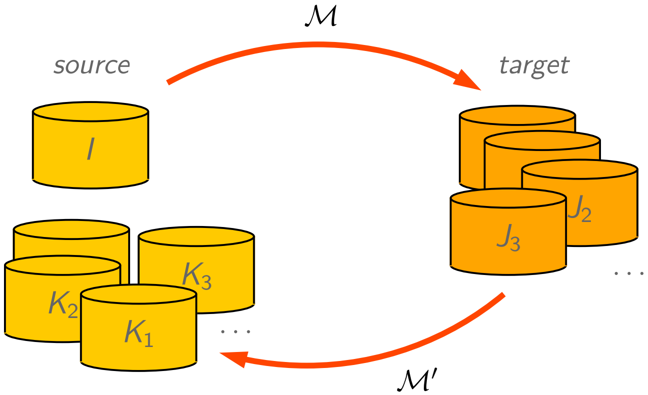

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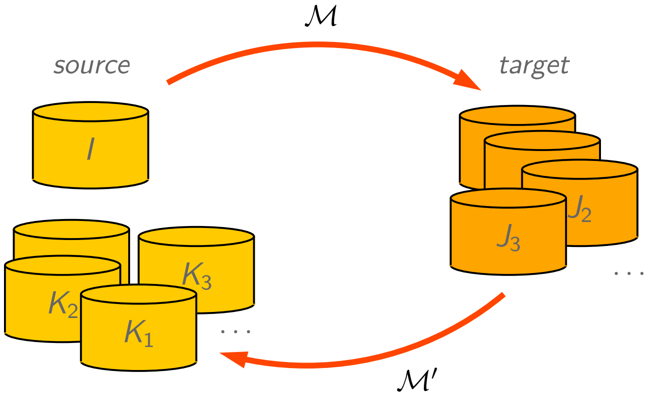

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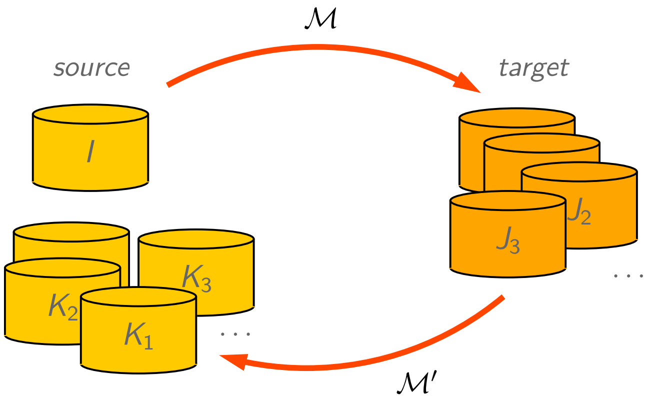

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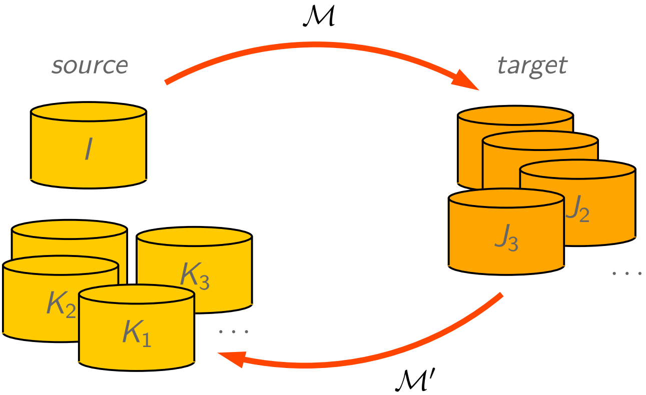

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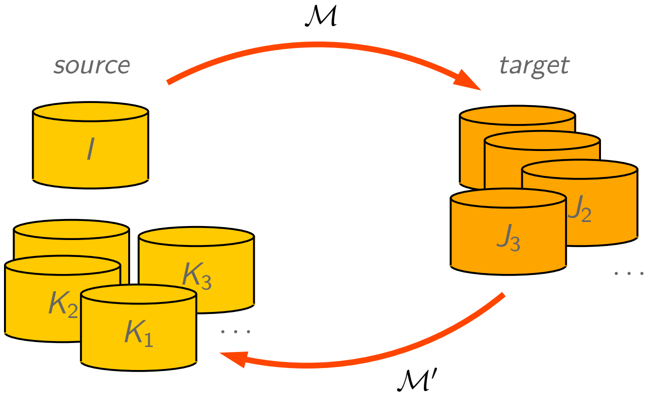

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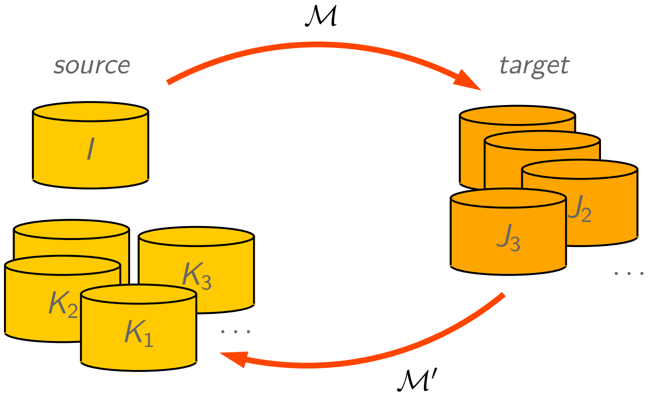

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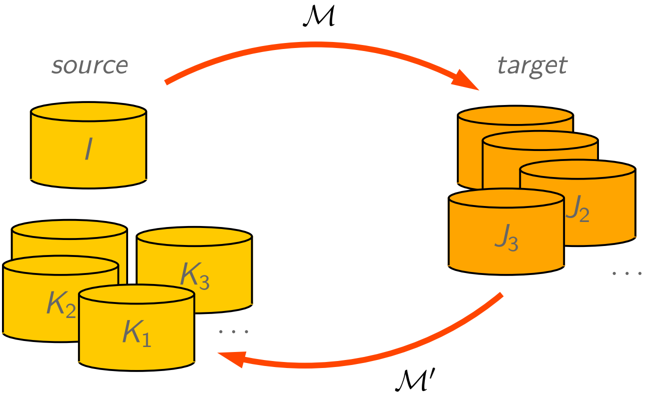

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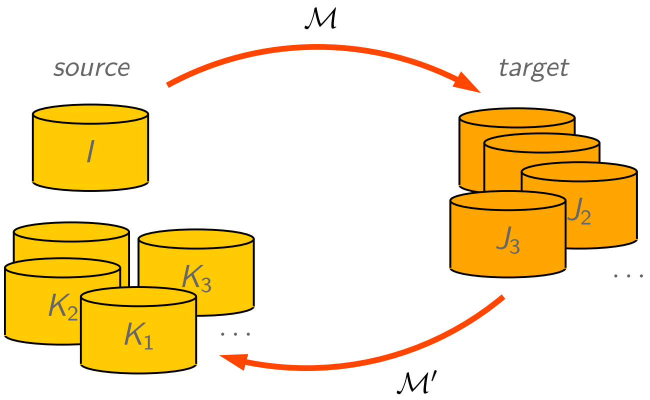

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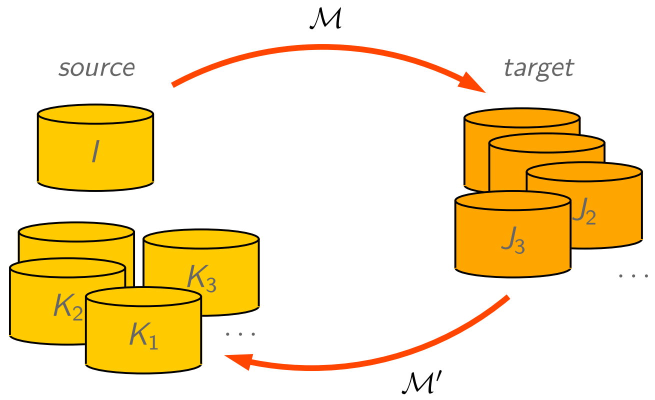

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For example:

- **All**-recovery: sound information for *all possible queries*
- **CQ**-recovery: sound information for *all conjunctive queries*

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We want a mapping such that the certain answers are *as close as possible* to $Q(I)$.

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$\mathcal{M}_1$ is better that any other possible $\mathcal{C}$-recovery!

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Theorem

- *If $\mathcal{M}$ has a Fagin-inverse, then:*

  $\mathcal{M}'$ is a Fagin-inverse of $\mathcal{M}$ iff
  $\mathcal{M}'$ is a $\mathbf{UCQ}^{\neq}$-maximum recovery of $\mathcal{M}$.

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- *If $\mathcal{M}$ has a quasi-inverse, then there exists a class of queries $\mathcal{C} \subseteq \mathbf{UCQ}^{\neq}$ such that:*

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If $\mathcal{M}'$ is a maximum recovery of $\mathcal{M}$
then $\mathcal{M}'$ is an **All**-maximum recovery of $\mathcal{M}$.

Why do we need another notion of inverse?

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- How do we exchange data using tgds with disjunctions?

We would like a *natural* notion of inverse such that:

- tgds always have an inverse, and
- such inverse can be expressed in a language with the same good properties as tgds for data exchange.

When focusing on **CQ** these issues can be solved

Main Theorem

Every mapping specified by tgds has a **CQ**-maximum recovery that can be specified by tgds with $\neq$ and $\mathbf{C}(\cdot)$ in the left-hand side.

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Main Theorem

Every mapping specified by tgds has a **CQ**-maximum recovery that can be specified by tgds with $\neq$ and $\mathbf{C}(\cdot)$ in the left-hand side.

Proof idea

We provide an algorithm to compute **CQ**-maximum recoveries of tgds that has as output a set of tgds$^{\neq,\mathbf{C}}$, and prove its correctness.

Query rewriting: a key concept in the algorithm

Query rewriting:

- Q' is a *rewriting* of Q under $\mathcal{M}$ if

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Query rewriting:

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Well-known result:

If $\mathcal{M}$ is specified by tgds, then for every query $Q \in \mathbf{CQ}$, there exists a query $Q' \in \mathbf{UCQ}^{=}$ that is a rewriting of Q under $\mathcal{M}$.

Step 1: compute a **CQ**-maximum recovery using rewriting of queries.

For a mapping $\mathcal{M}$ specified by tgds, compute $\mathcal{M}'$ as follows:

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Lemma

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Lemma

*$\mathcal{M}'$ is a **CQ**-maximum recovery of $\mathcal{M}$.*

Problem: disjunctions and equalities in the right-hand side.

Step 2: eliminate right-hand side equalities

Example:

$$A(x,y) \quad \rightarrow \quad R(x,y) \quad \vee \quad (P(x) \wedge x = y) \quad \vee \quad S(x) \wedge T(y)$$

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Step 2: eliminate right-hand side equalities

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- Let $\mathcal{M}'$ be the mapping constructed in Step 1.
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Problem: formulas still have disjunctions in the right-hand side.

Key concept in Step 3:
Cartesian product of queries (intuition)

$$\begin{array}{rcr} Q_1(x_1, x_2) & : & T(x_1, x_2) \wedge R(x_1, x_1) \\ Q_2(x_1, x_2) & : & \exists y \;\; T(x_1, y) \wedge R(x_2, x_2) \end{array}$$

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If we know that tuple (a, b) is an answer to one of the two queries

What can we *certainly infer* about tables T and R?

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Key concept in Step 3: *Cartesian product* of queries (formalization)

Definition

Homomorphism between conjunctive queries: function h that

- maps existential variables to free or existential variables
- is the identity over free variables

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The conjuntive query Q is the *Cartesian product* of Q_1 and Q_2 if it is the *closest query* to both Q_1 and Q_2 in terms of homomorphism.

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In the paper we give an algorithm for the Cartesian product:

- a simple extension of the Cartesian product of graphs.

Step 3: eliminate disjunctions

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- Let $\mathcal{M}''$ be the mapping constructed in Step 2.
- Construct $\mathcal{M}^\star$ by replacing every dependency

$$\varphi(\bar{x}) \;\rightarrow\; \beta_1(\bar{x}) \;\vee\; \beta_2(\bar{x}) \;\vee\; \cdots \;\vee\; \beta_n(\bar{x})$$

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$\mathcal{M}^\star$ is a **CQ***-maximum recovery of $\mathcal{M}$.*

Summing up...

Algorithm

Let $\mathcal{M}$ be a mapping specified by tgds:

1. Compute a **CQ**-maximum recovery by using rewriting.
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Highlights of the algorithm:

- We use query-rewriting to compute **CQ**-maximum recoveries:

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Highlights of the algorithm:

- We use query-rewriting to compute **CQ**-maximum recoveries:
  - exponential-time worst case, but
  - we can reuse the large body of work on query-rewriting.

The language of **CQ**-maximum recoveries

Theorem

The language of tgds$^{\neq,\mathbf{C}}$ *is the* minimal language *needed to specify* **CQ***-maximum recoveries of tgds.*

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The language has the same good properties as tgds, in particular:

- the *chase* procedure can be used to exchange data,
- a *canonical universal solution* (and a *core*) exists for every source instance.

Concluding remarks

- A new notion of inverse of schema mappings based on queries and certain answers
- Previously proposed notions of inverse can be obtained by considering specific query languages.
- When focusing on **CQ** some practical issues are solved, in particular:

  Every mapping specified by tgds has a
  CQ-maximum recovery specified in a language
  with the same good properties as tgds for data exchange.

Inverting Schema Mappings: Bridging the Gap between Theory and Practice

Marcelo Arenas, Jorge Pérez, Cristian Riveros, Juan Reutter

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