Static Analysis and Optimization of Semantic Web Queries

Andrés Letelier Jorge Pérez Reinhard Pichler Sebastian Skritek

PUC Chile Universidad de Chile TU Vienna

RDF data model for the Semantic Web

SPARQL query language for RDF (W3C initiatives)

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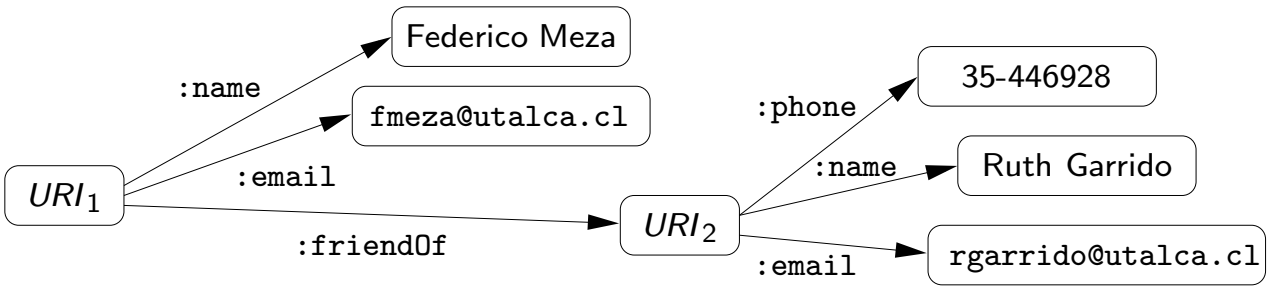

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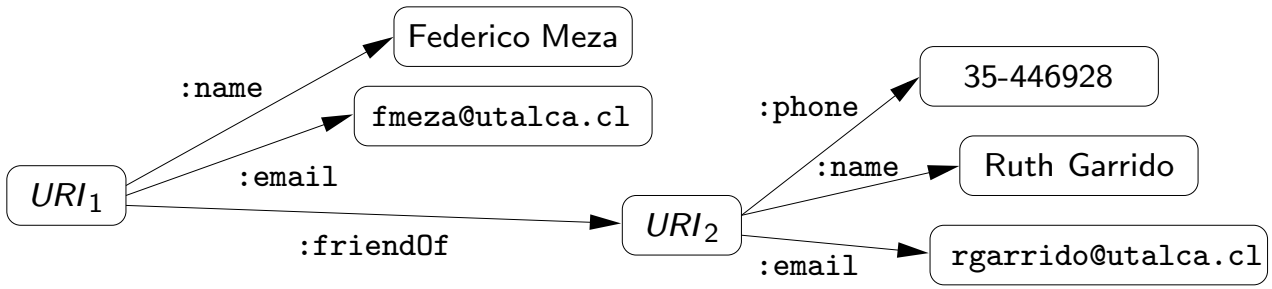

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SELECT ?N ?E
WHERE
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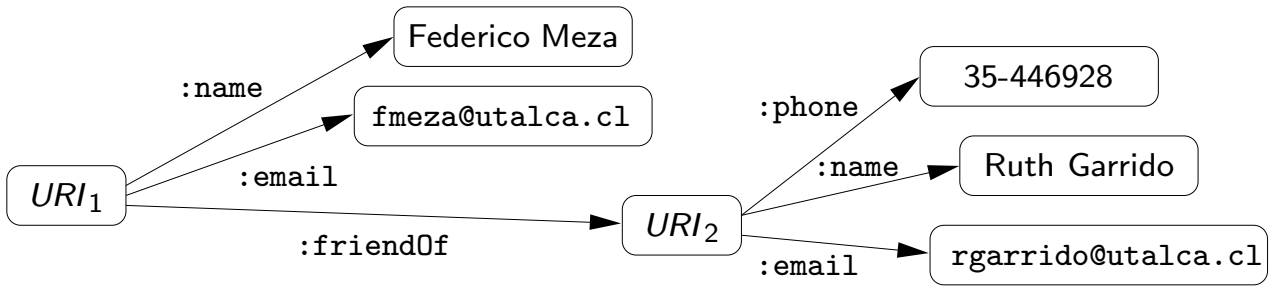

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SELECT ?N ?E
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{
  ?X :name ?N .
  ?X :email ?E .
  ?X :friendOf ?Y . ?Y :name "Ruth Garrido"
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- Optimization rules have been proposed

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Our goals:

- Static analysis of SPARQL
- Apply CQ tractable results to SPARQL

Main contributuions

Conceptual contribution: *Pattern trees*

- Tree representation for a fragment of SPARQL graph patterns
- First step towards an algebra for SPARQL *query plans*

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Technical contributions:

- Homomorphism-based redundancy elimination and equivalence
- Complexity of testing equivalence (NP) and subsumption (Π_2^P)
- Tractable enumeration & complexity of counting

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Outline

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SPARQL: a simple RDF query language

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We focus on P

(i.e. we do not consider projection)

Graph patterns: interesting features of pattern matching on graphs

- Conjunctions

```
{ P1 .
  P2 }
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- Grouping

```
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    P2 }

  { P3 .
    P4 }


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- Optional parts

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    P2
    OPTIONAL { P5 }  }
  { P3 .
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Graph patterns: interesting features of pattern matching on graphs

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{ P9 }
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Graph patterns: interesting features of pattern matching on graphs

- Conjunctions
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Graph patterns: interesting features of pattern matching on graphs

- Conjunctions
- Grouping
- Optional parts
- Nesting of optionals
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- Filtering
- ...
- new version: regular expressions, federated queries, sub-queries, etc.

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A standard algebraic syntax

- Triple patterns: just triples + variables ($?X, ?Y, \ldots$)

original SPARQL syntax	algebraic syntax
`?X :name "john"`	$(?X, \text{name}, \text{john})$

- Graph patterns: algebraic expressions over triple patterns

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`{ P1 . P2 }`	$(P_1 \text{ AND } P_2)$
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Evaluating SPARQL patterns: a simple example

Evaluation formalized in terms of *(partial) mappings*

$$\mu : \text{Variables} \longrightarrow \text{RDF-Terms}$$

Example

$(R_1, \text{name}, \text{john})$	$(R_2, \text{name}, \text{paul})$	$(R_3, \text{name}, \text{ringo})$
$(R_1, \text{email}, \text{J@ed.ex})$		$(R_3, \text{email}, \text{R@ed.ex})$
		$(R_3, \text{wPage}, \text{w3.rin.ed})$

$$((?X, \text{name}, ?Y) \text{ OPT } (?X, \text{wPage}, ?W))$$

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if a variable occurs

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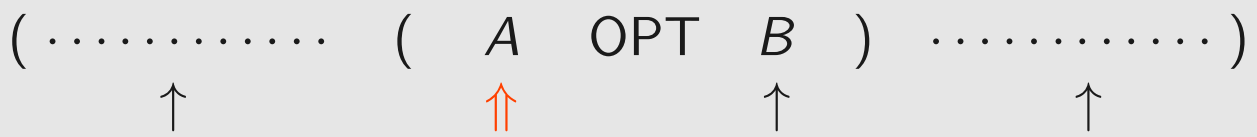

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$$\big(\; (?X, \text{name}, \text{john}) \; \text{OPT} \; \big[(?Y, \text{name}, \text{ringo}) \; \text{OPT} \; (?X, \text{wPage}, ?W) \big] \; \big)$$

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[PAG09] evaluation for the well-designed fragment is *coNP*

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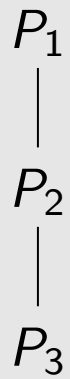
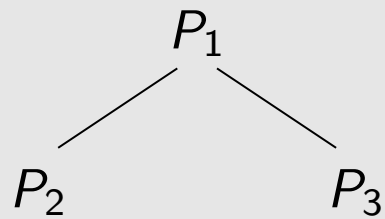
Pattern Trees

Pattern tree example

Intuition





$$\Big(P_1 \text{ OPT } (P_2 \text{ OPT } P_3)\Big) \qquad \Big((P_1 \text{ OPT } P_2) \text{ OPT } P_3\Big)$$

Quasi-well designed pattern trees (QWDPTs)

Definition

A pattern tree is *quasi-well desgined* if for every two nodes mentioning $?X$, there is a common ancestor that also mentions $?X$

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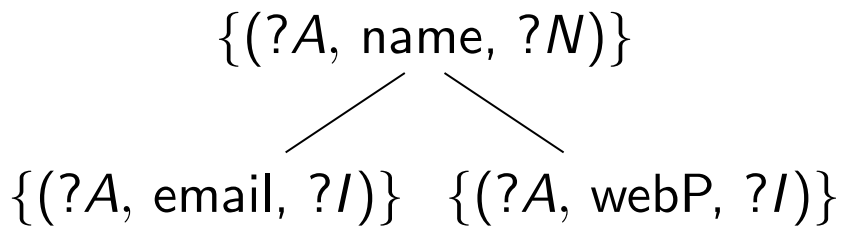

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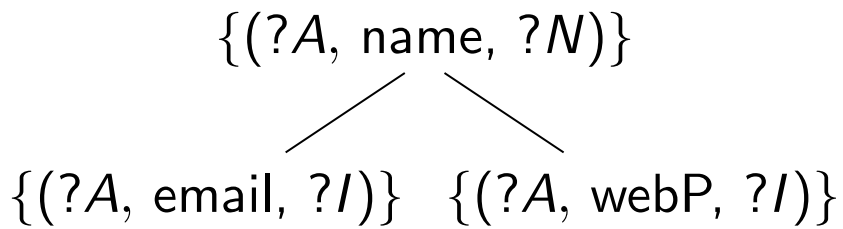

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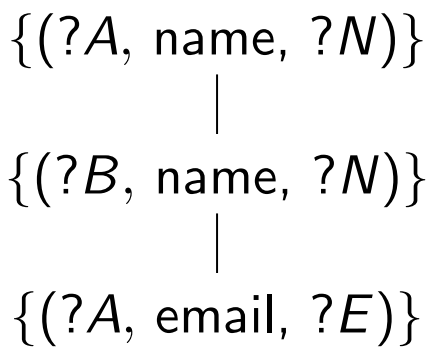
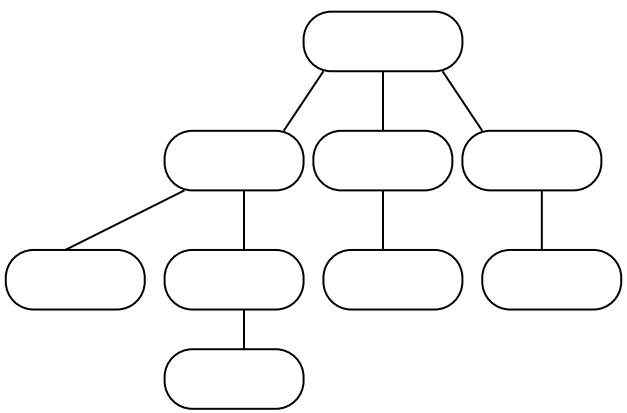


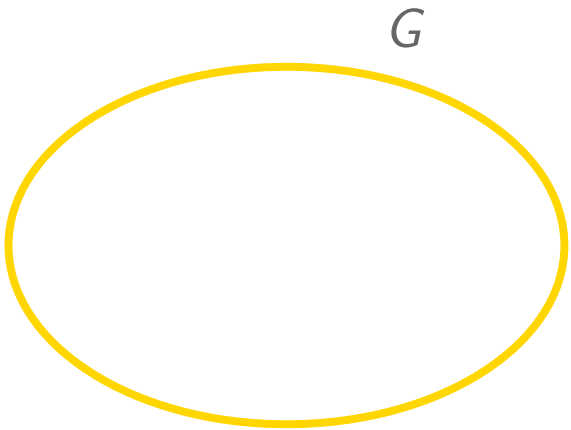
QWDPT

Top-down evaluation of pattern trees

QWDPTs allow for a *natural evaluation* from the root to the leaves:

- compute a mapping for a given node
- then try to extend it with the evaluation of its children
- stop in a branch if no further extension is possible





Top-down evaluation of pattern trees

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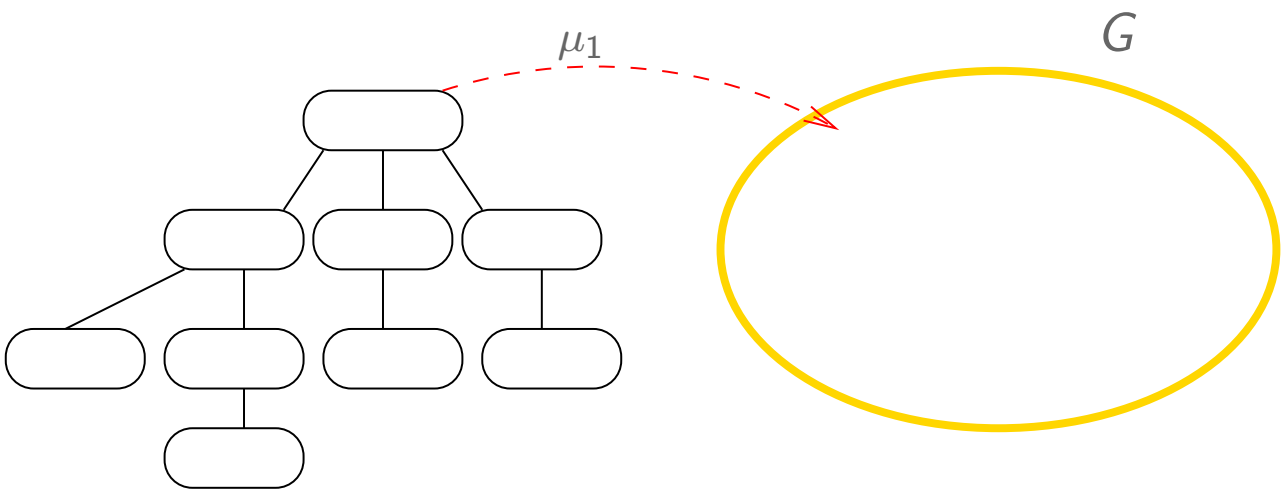

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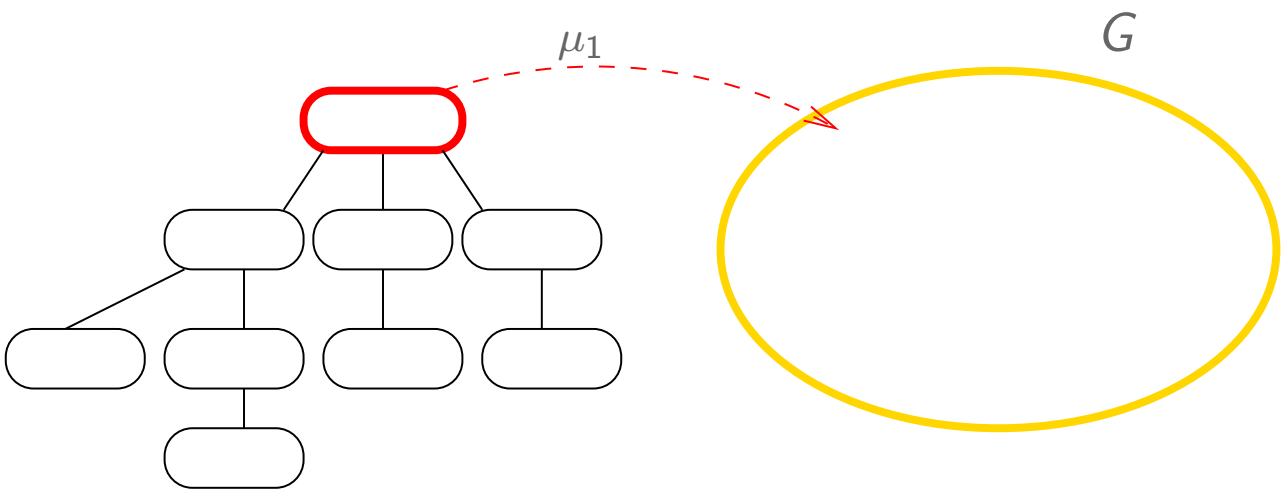

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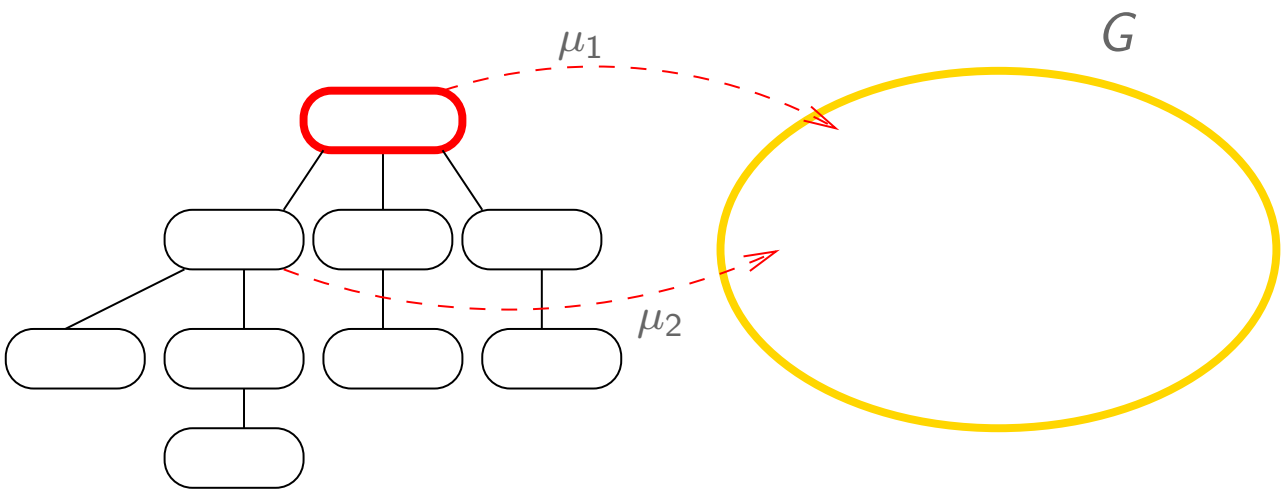

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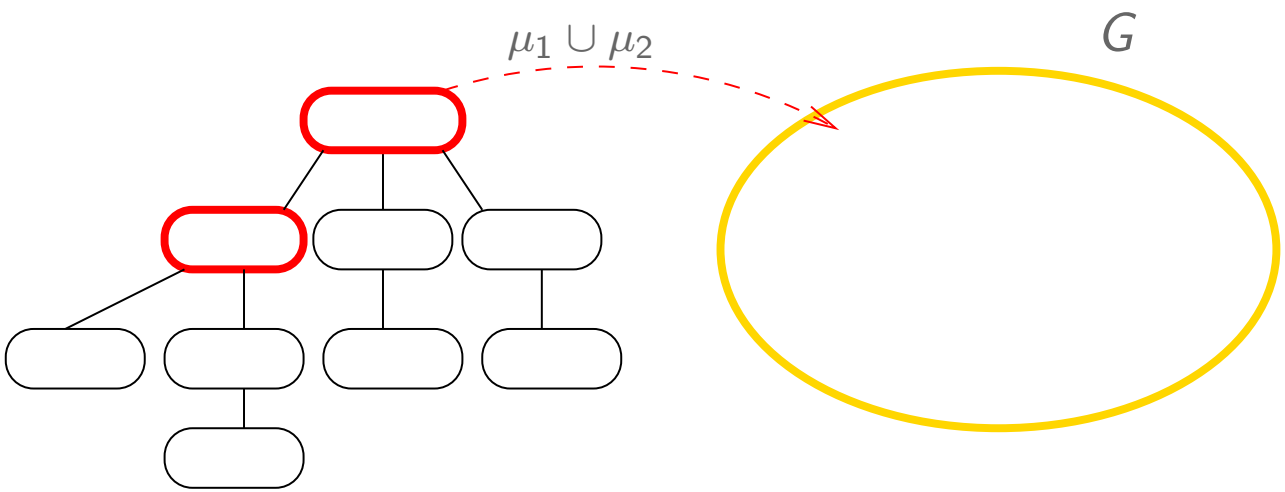

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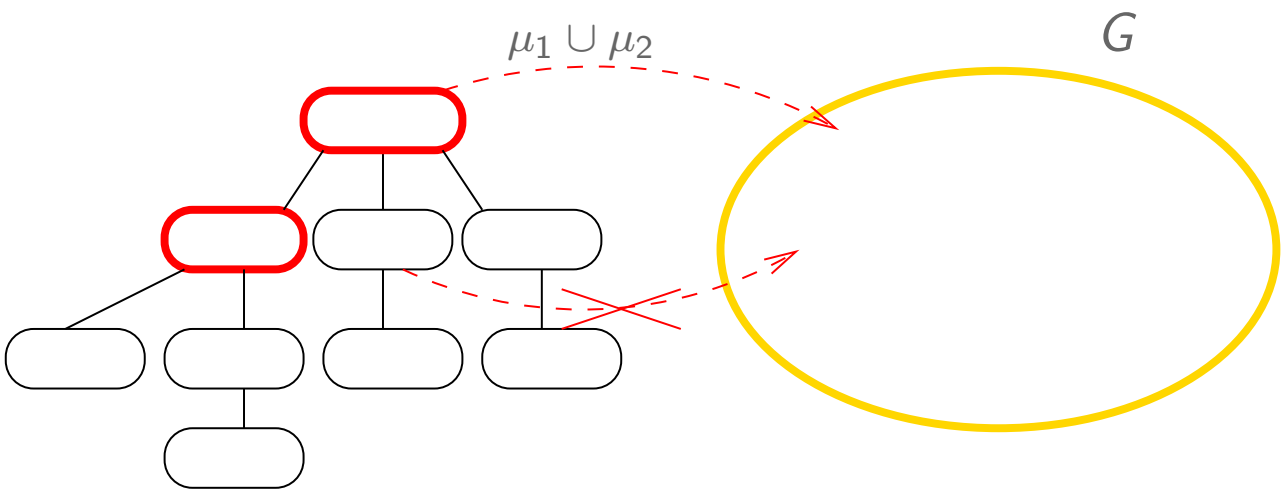

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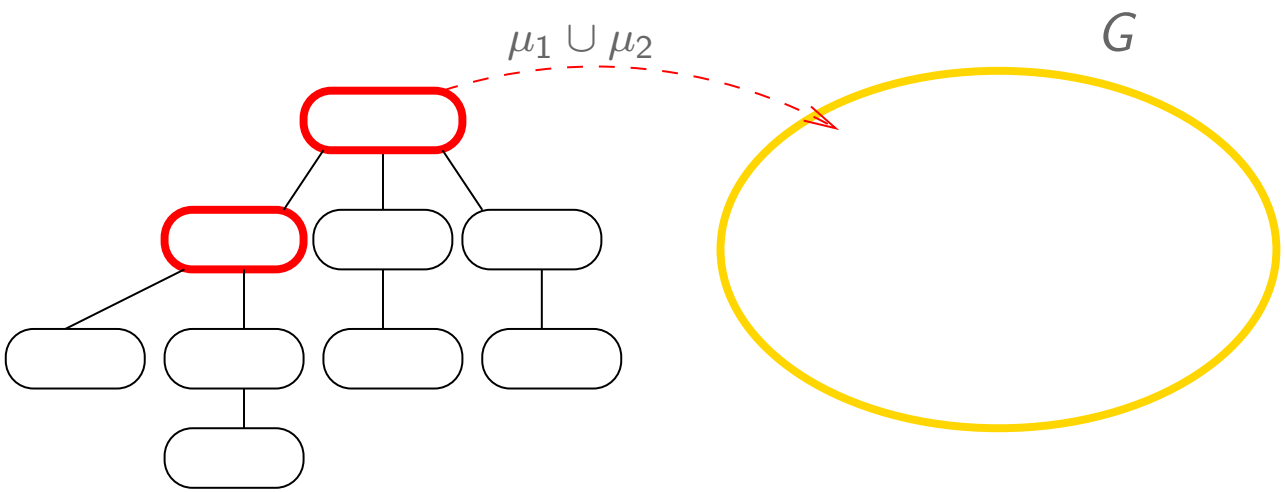

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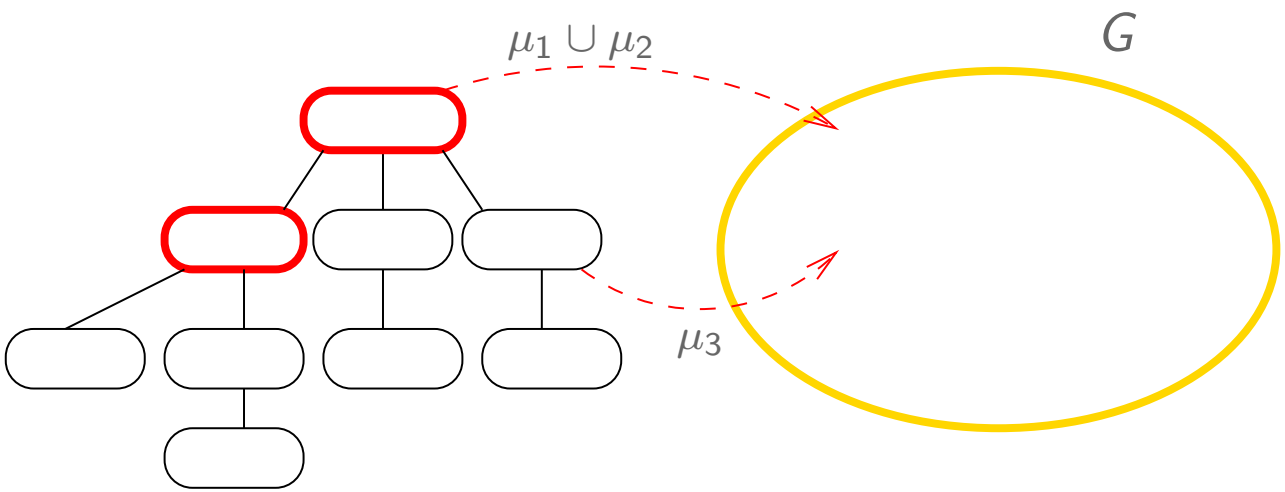

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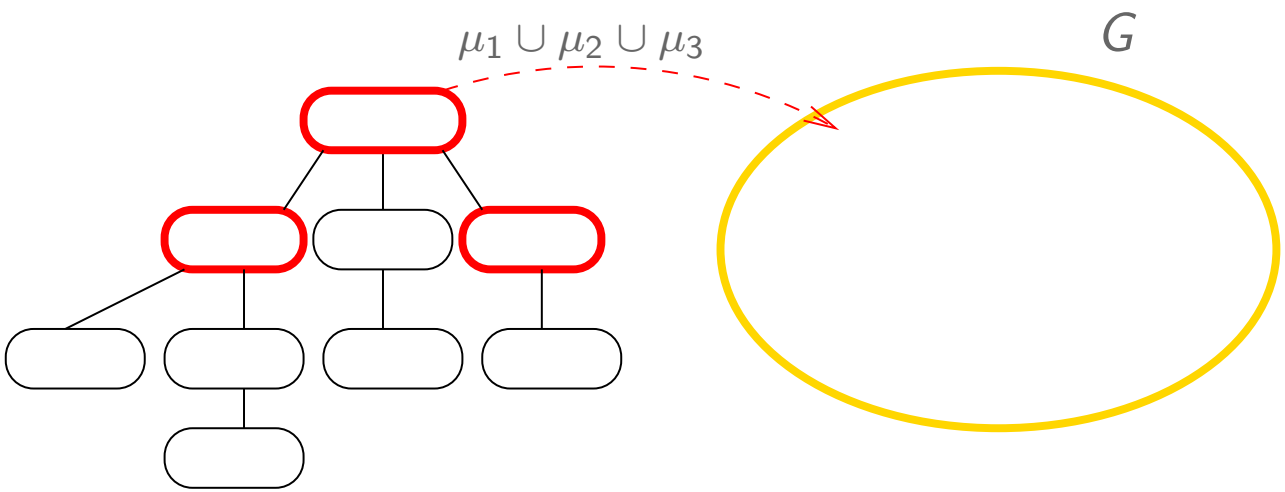

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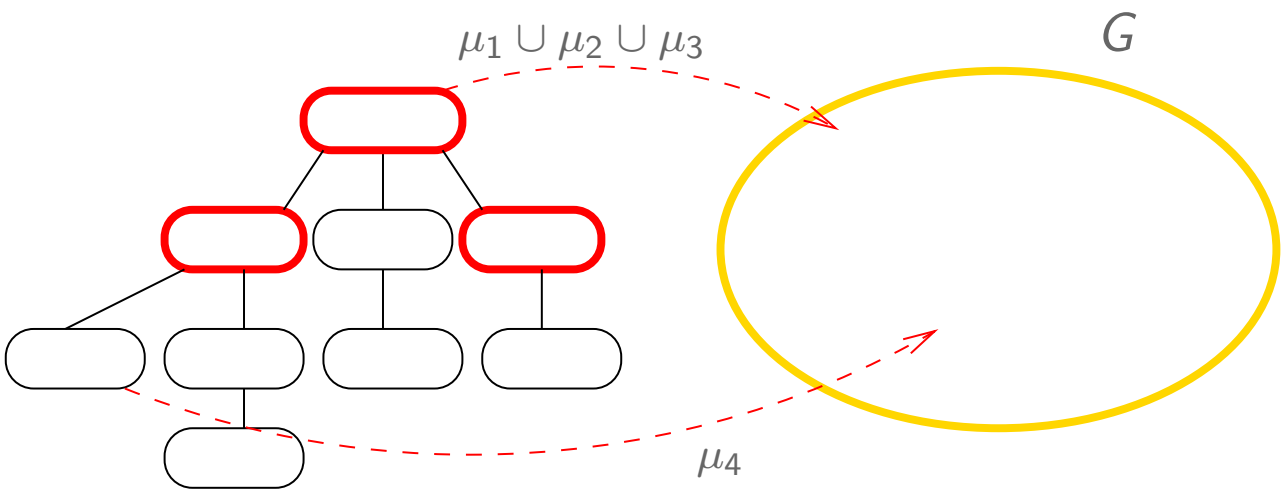

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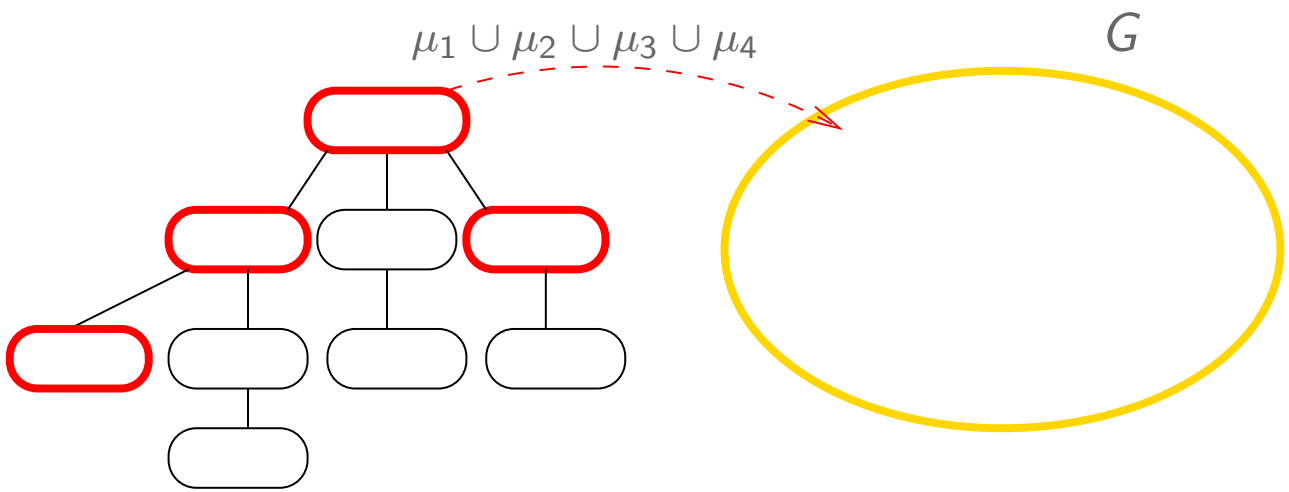

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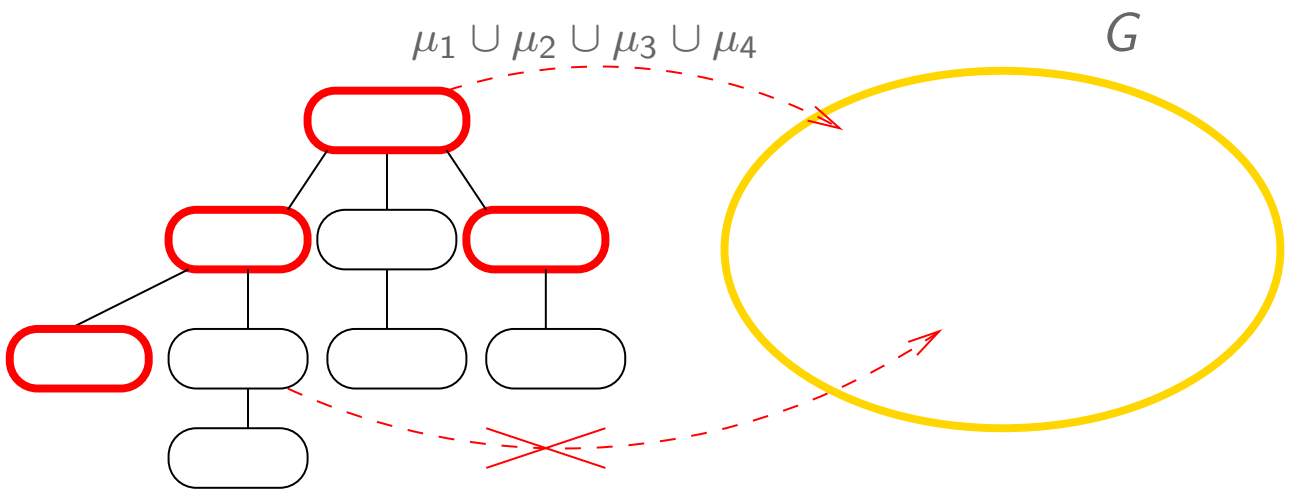

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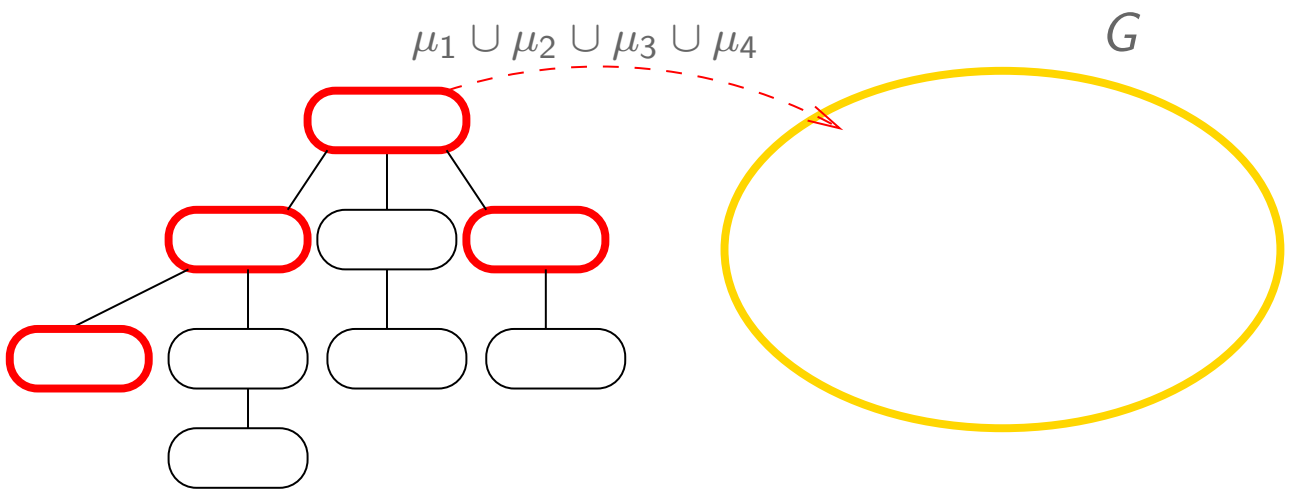

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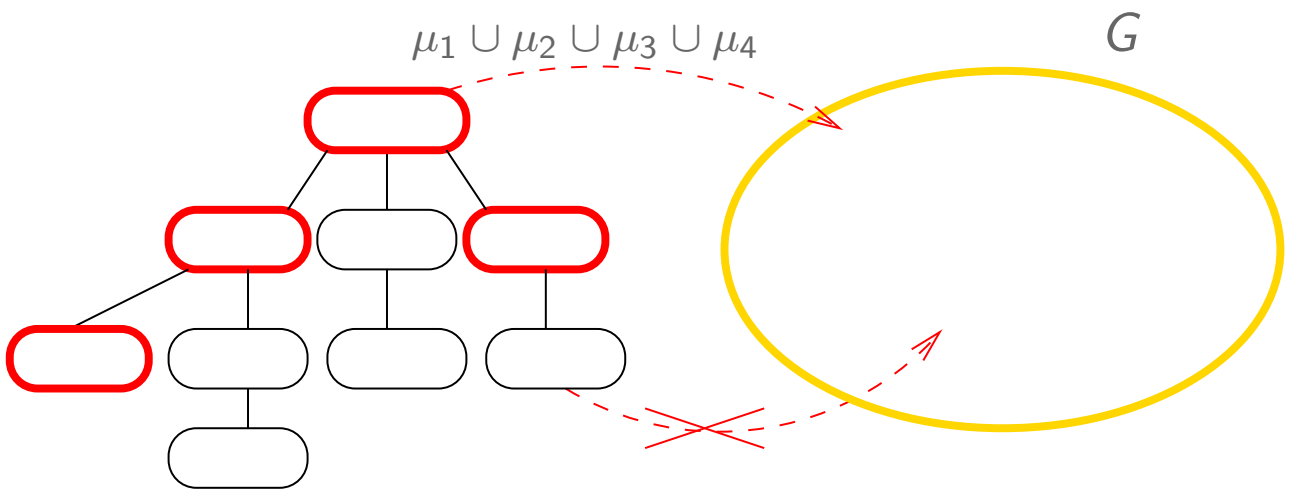

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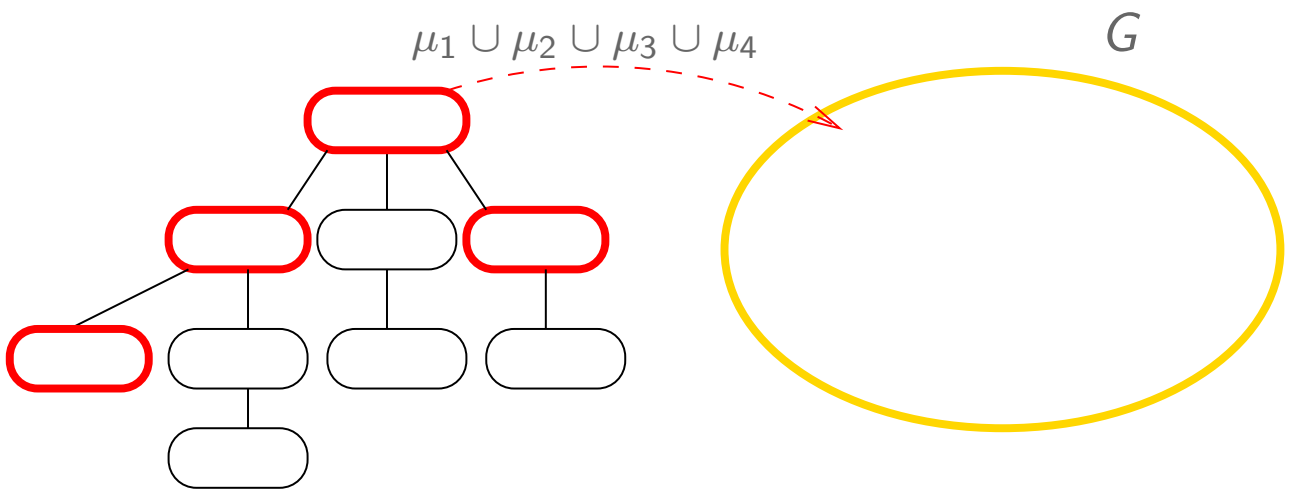

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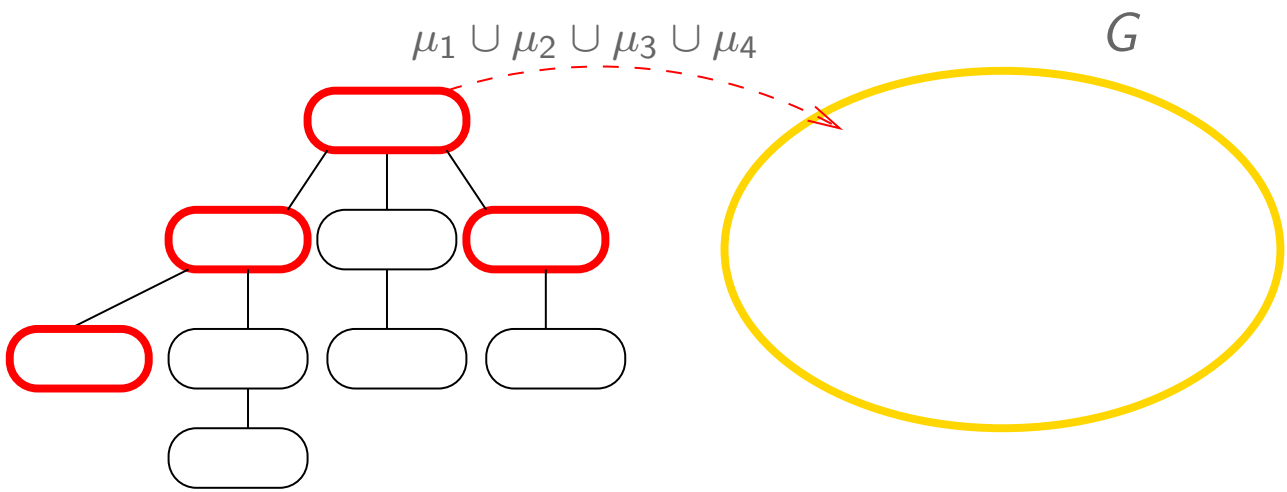

$\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4$ is a solution over G

QWDPTs represent well-designed SPARQL graph patterns

Theorem

For every well-designed pattern P there exists a QWDPT $\mathcal{T}$ s.t.

$$\textit{top-down evaluation of } \mathcal{T} \equiv \textit{algebraic evaluation of } P$$

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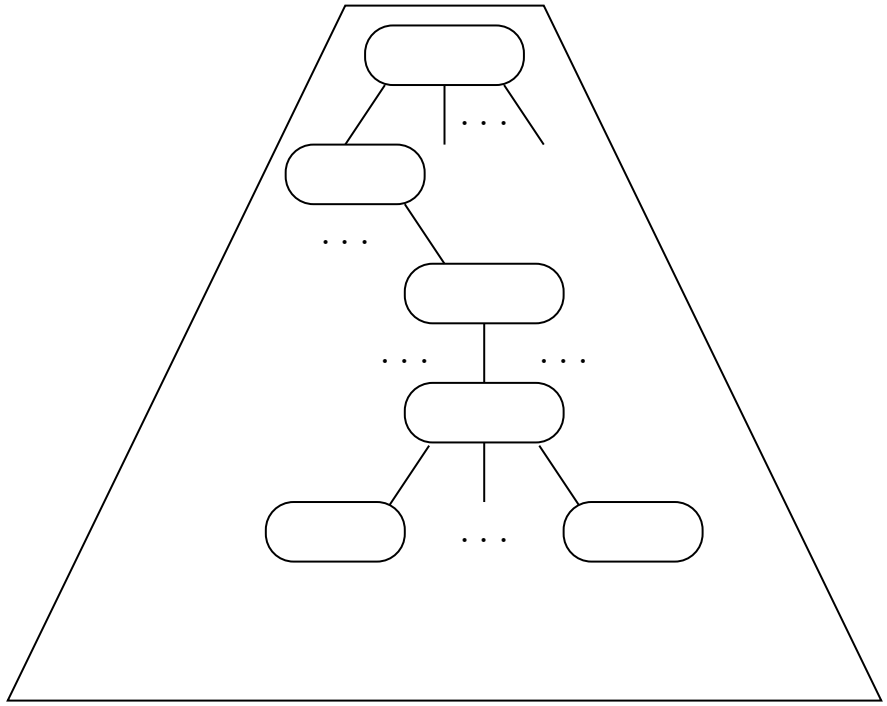
Moreover, we can go from one representation
into the other in polynomial time

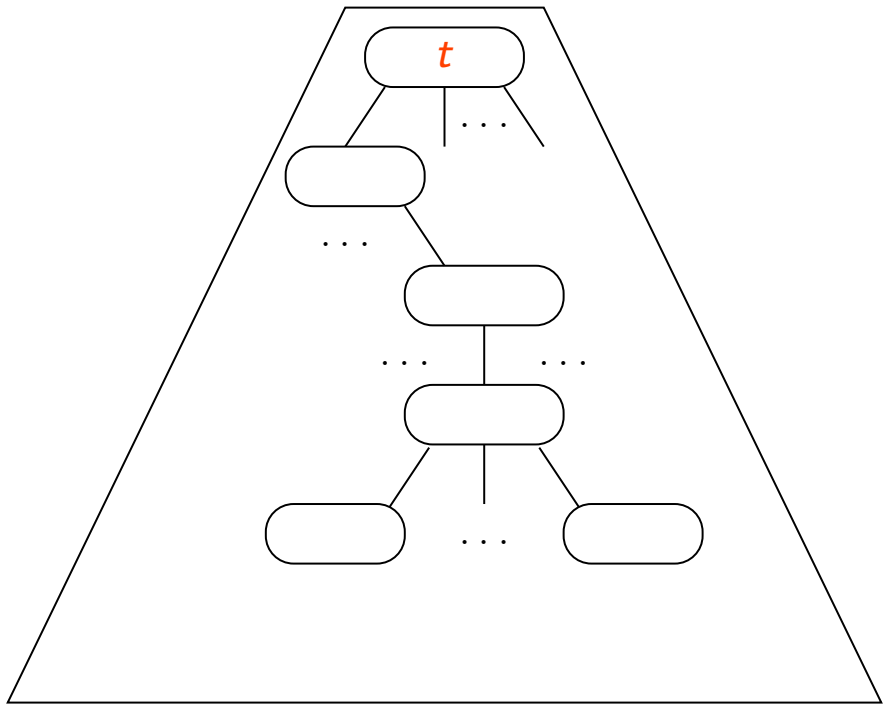
Outline

Transformation rules: deleting redundant triples



R1: If a triple pattern t belongs to node n

Transformation rules: deleting redundant triples



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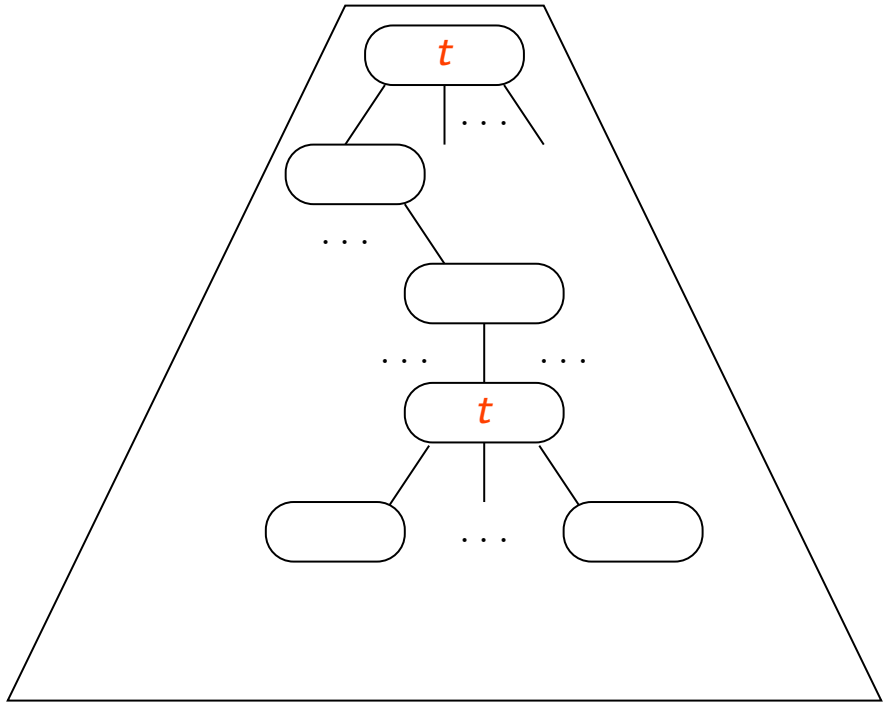

R1: If a triple pattern t belongs to node n and to a descendant n' of n,

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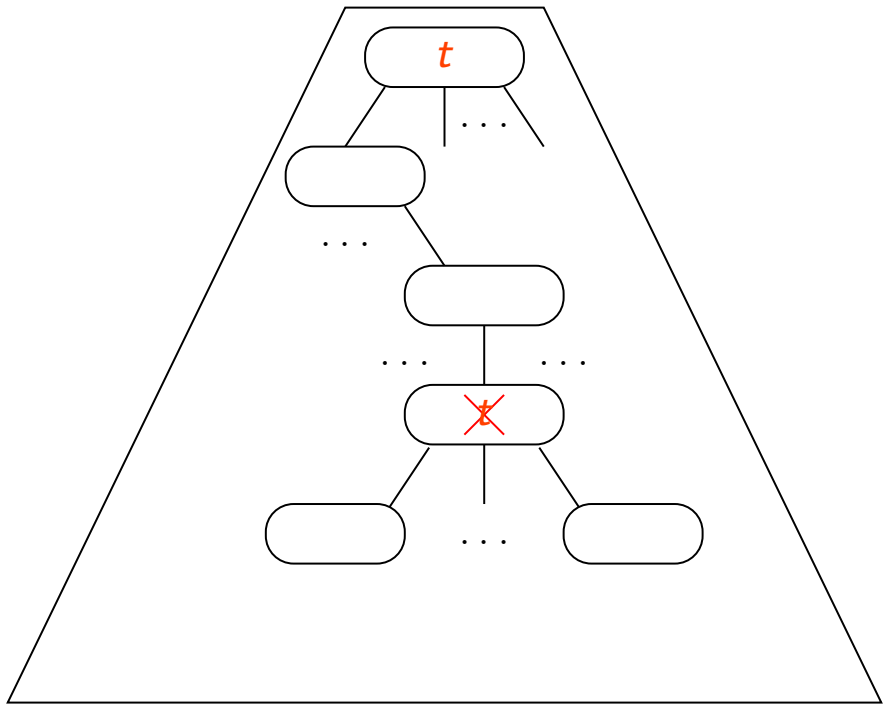
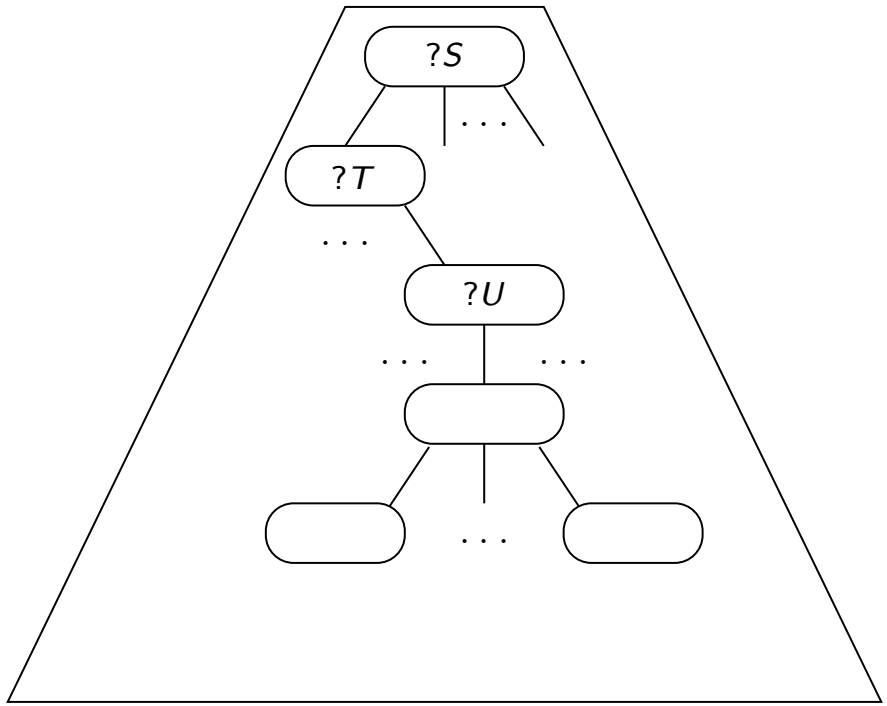


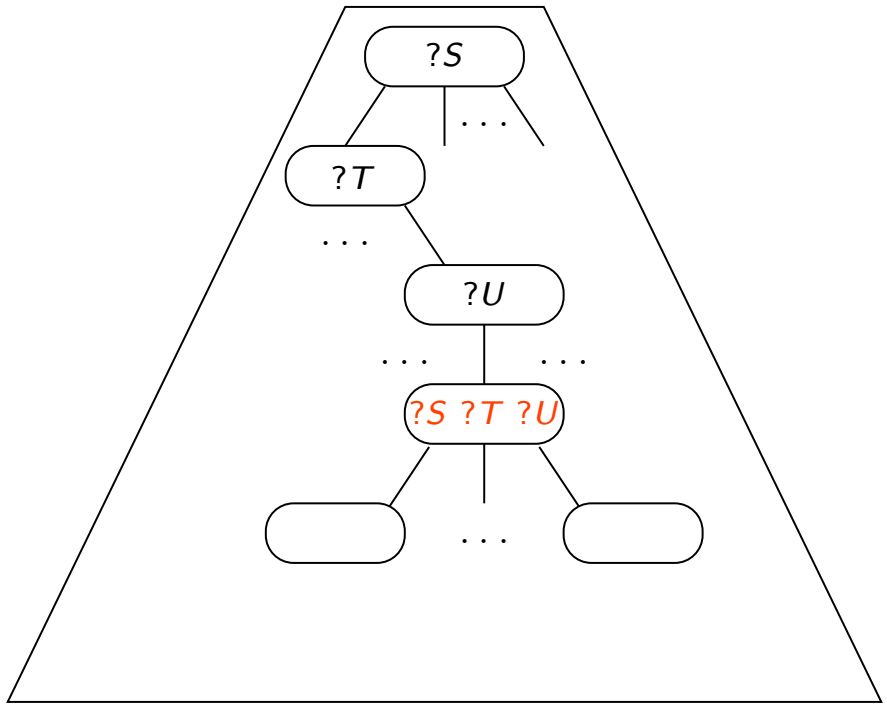
R1: If a triple pattern t belongs to node n and to a descendant n' of n, then delete t from n'.

Transformation rules: deleting redundant nodes



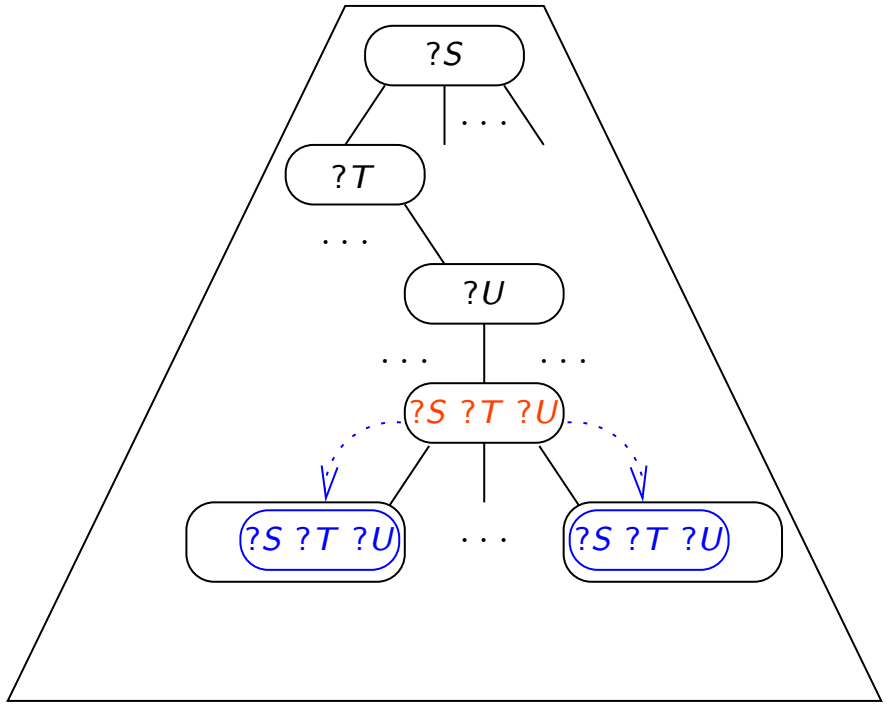
R2: If node n

Transformation rules: deleting redundant nodes



R2: If node n does not introduce any new variable w.r.t. its ancestors,

Transformation rules: deleting redundant nodes



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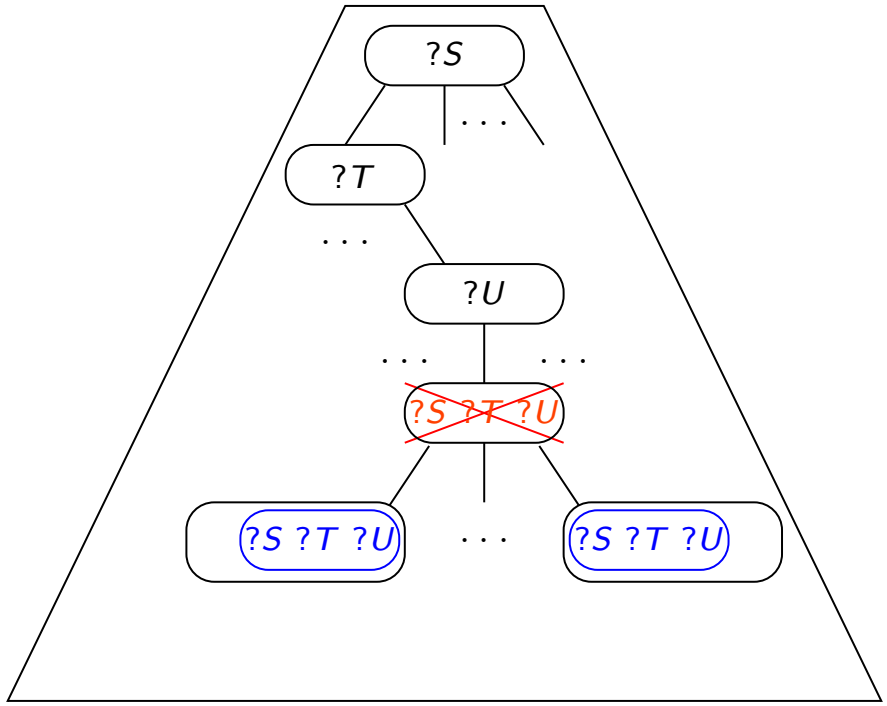

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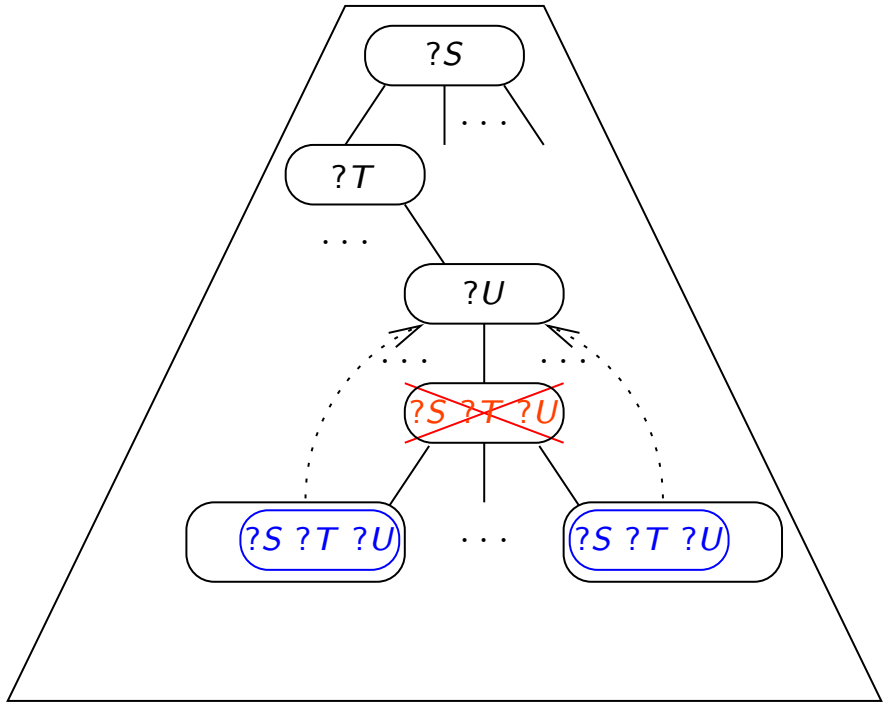

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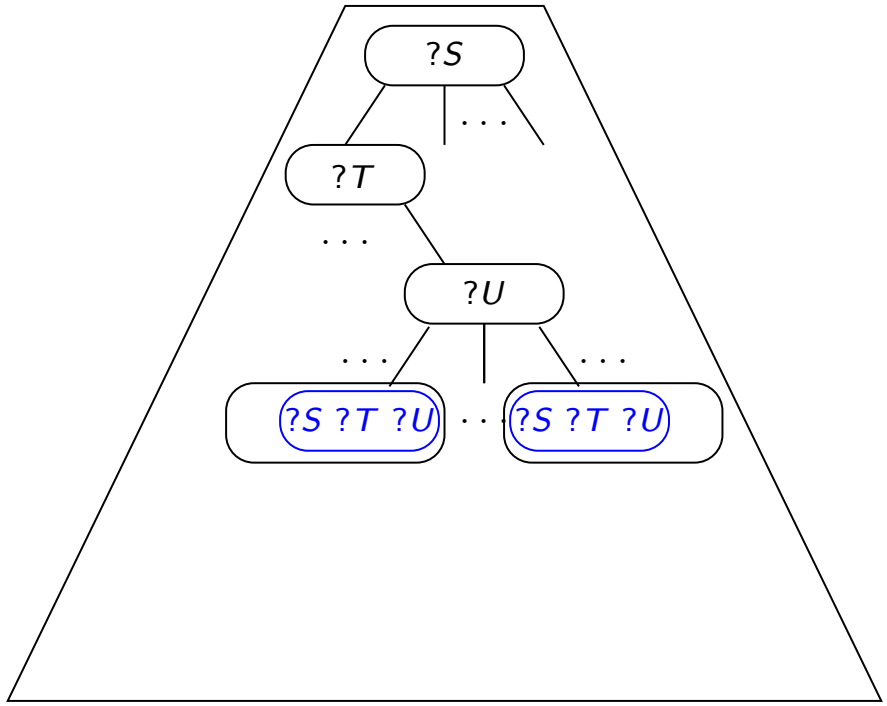
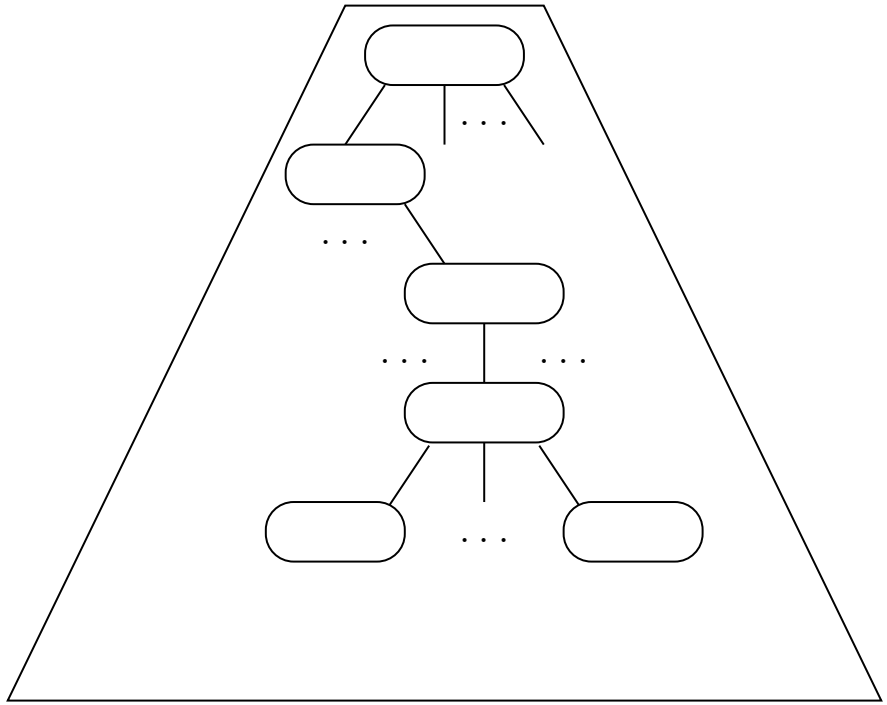


R2: If node n does not introduce any new variable w.r.t. its ancestors, then *push* copies of n into its children, and delete the node.

Transformation rules: homomorphism upwards



R3: If there is a *homomorphism*

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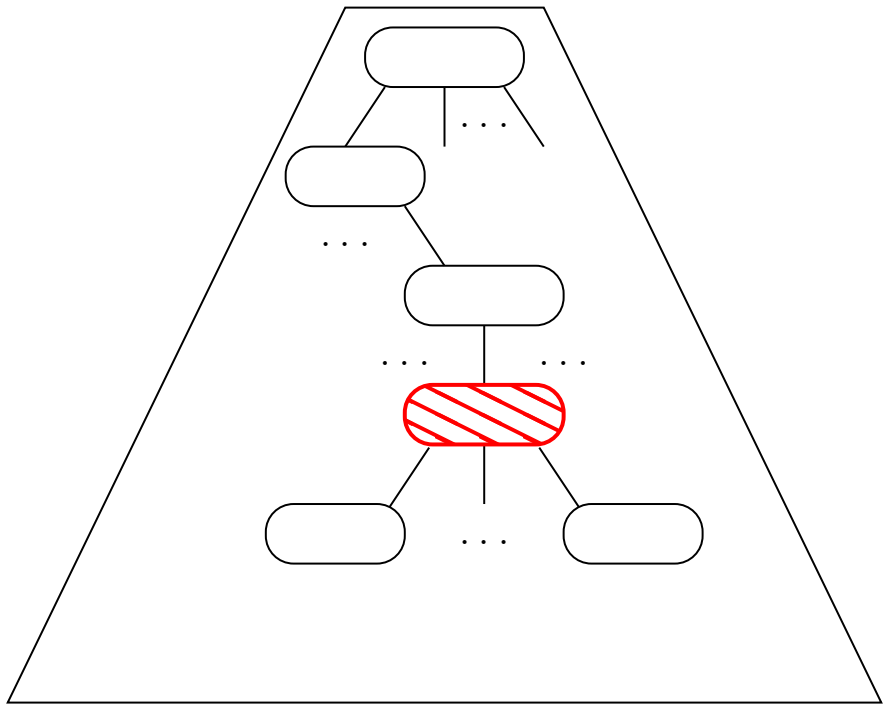

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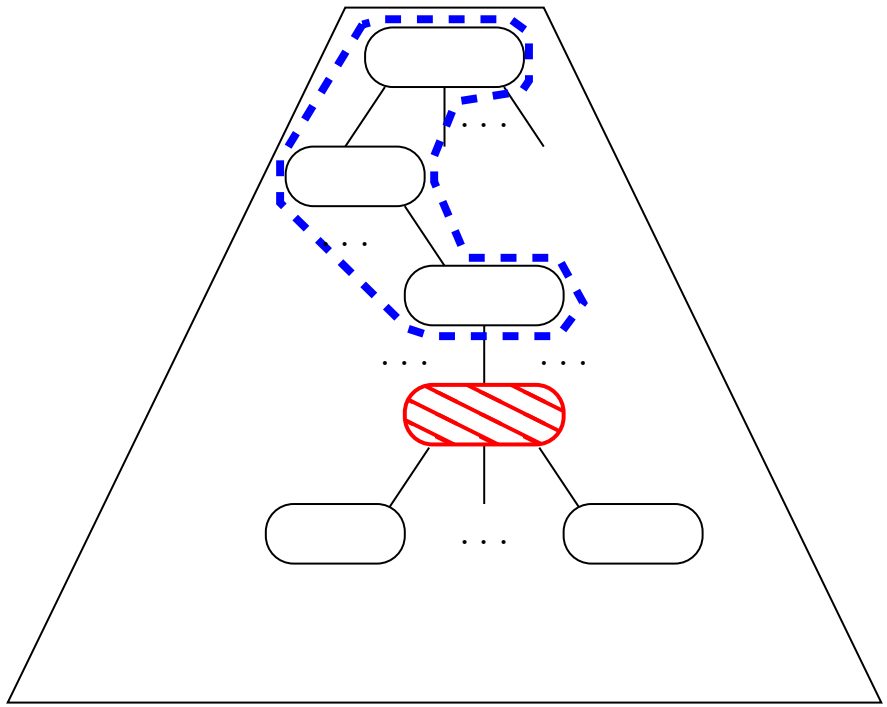

R3: If there is a *homomorphism* from node n to the branch of ancestors of n,

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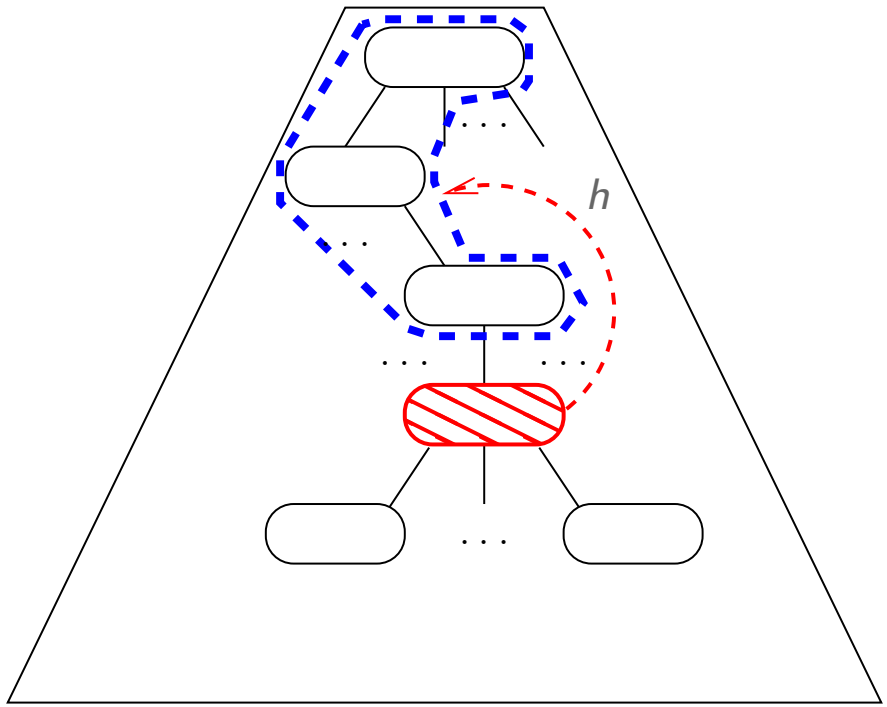

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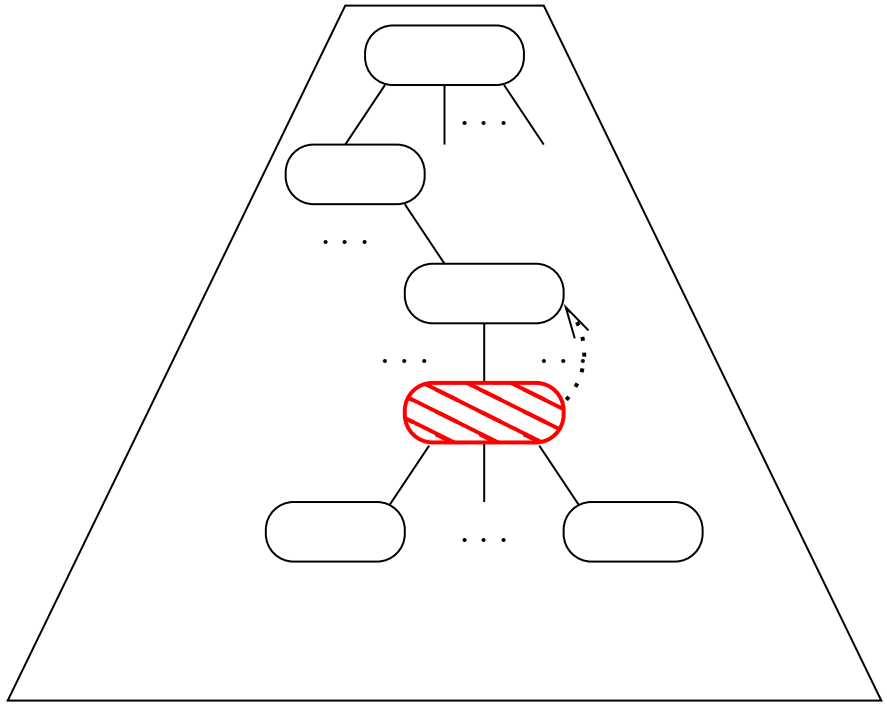

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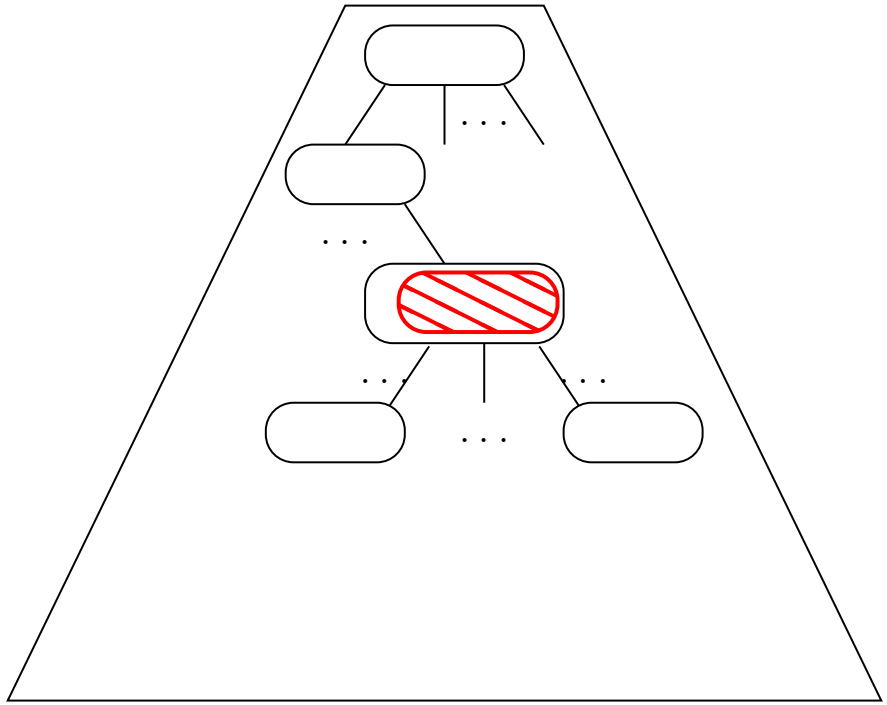

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Normal forms

NR-normal-form: Apply R1 and R2 in any order

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Impact for optimization

Rules application *eliminate optional parts*

Also useful when testing equivalence of QWDPT

Outline

Subsumption

Definition

Mapping μ_1 is *subsumed* by μ_2, denoted by $\mu_1 \sqsubseteq \mu_2$, if μ_2 extends μ_1 with more variable bindings

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$\mathcal{T}_1$ is *subsumed* by $\mathcal{T}_2$, denoted by $\mathcal{T}_1 \sqsubseteq \mathcal{T}_2$, if for ever G

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$\sqsubseteq$ is a natural notion of *containment* for SPARQL

$$(?X, \text{name}, ?N) \sqsubseteq ((?X, \text{name}, ?N) \text{ OPT } (?X, \text{email}, ?E))$$

Subsumption is Π_2^P-complete

Theorem

$\mathcal{T}_1 \sqsubseteq \mathcal{T}_2$ iff for every subtree $\mathcal{T}_1'$ of $\mathcal{T}_1$ there exists a subtree $\mathcal{T}_2'$ of $\mathcal{T}_2$, and a homomorphism from triples in $\mathcal{T}_2'$ to triples in $\mathcal{T}_1'$ (which is the identity over variables in $\mathcal{T}_1'$)

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Theorem

Checking subsumption of QWDPTs is Π_2^P-complete

Equivalence & subsumption

Pattern trees $\mathcal{T}_1$ and $\mathcal{T}_2$ are equivalent, denoted by $\mathcal{T}_1 \equiv \mathcal{T}_2$ if they give the same result evaluated over every RDF graph

Proposition

For QWDPTs $\mathcal{T}_1$ and $\mathcal{T}_2$

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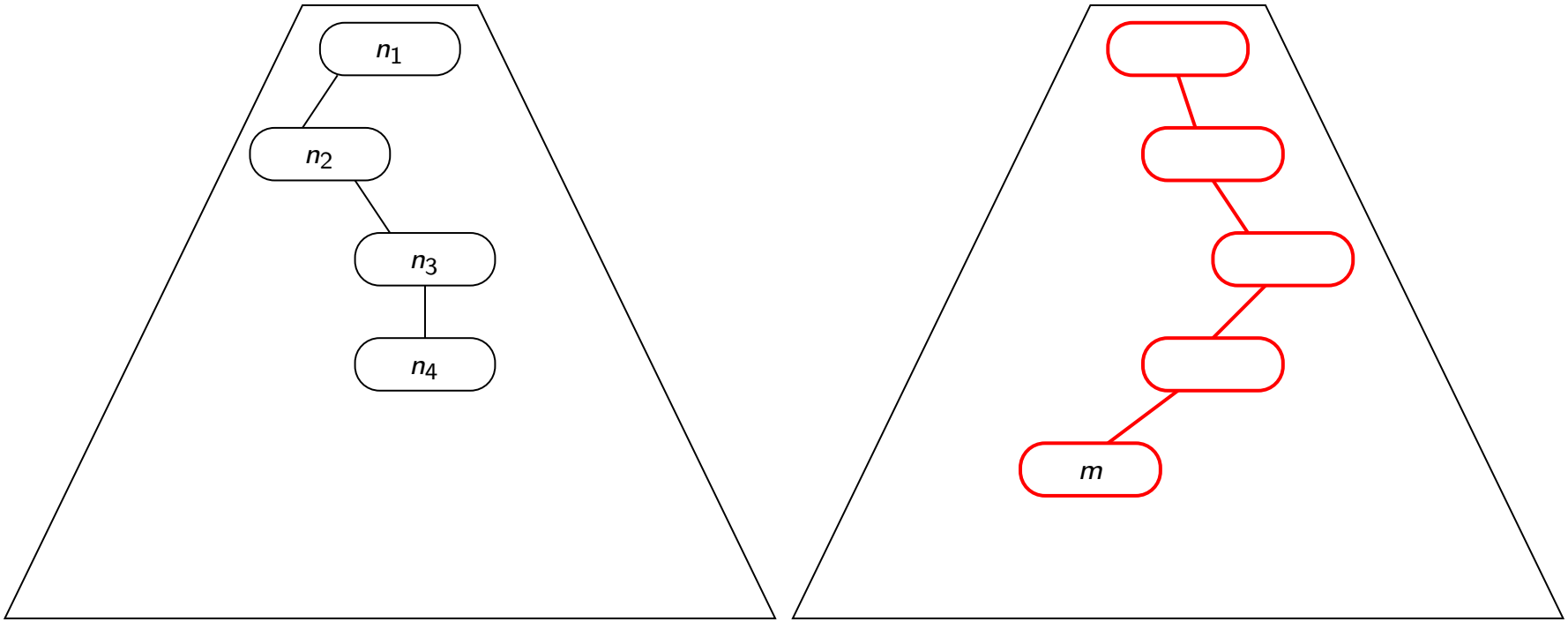
This gives us with a Π_2^P equivalence test, but we can do better!

Strong homomorphisms: key concept for equivalence test

Strong homomorphism among branches: a set of homomorphisms



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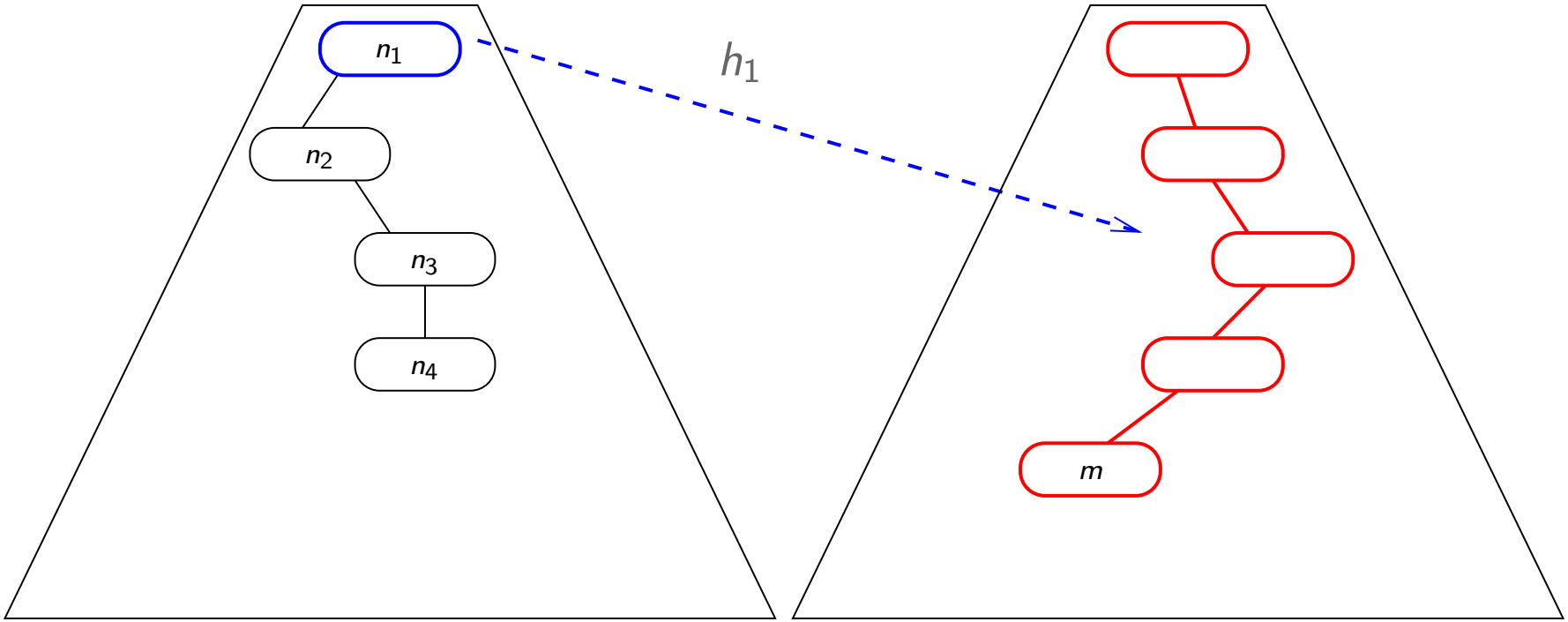

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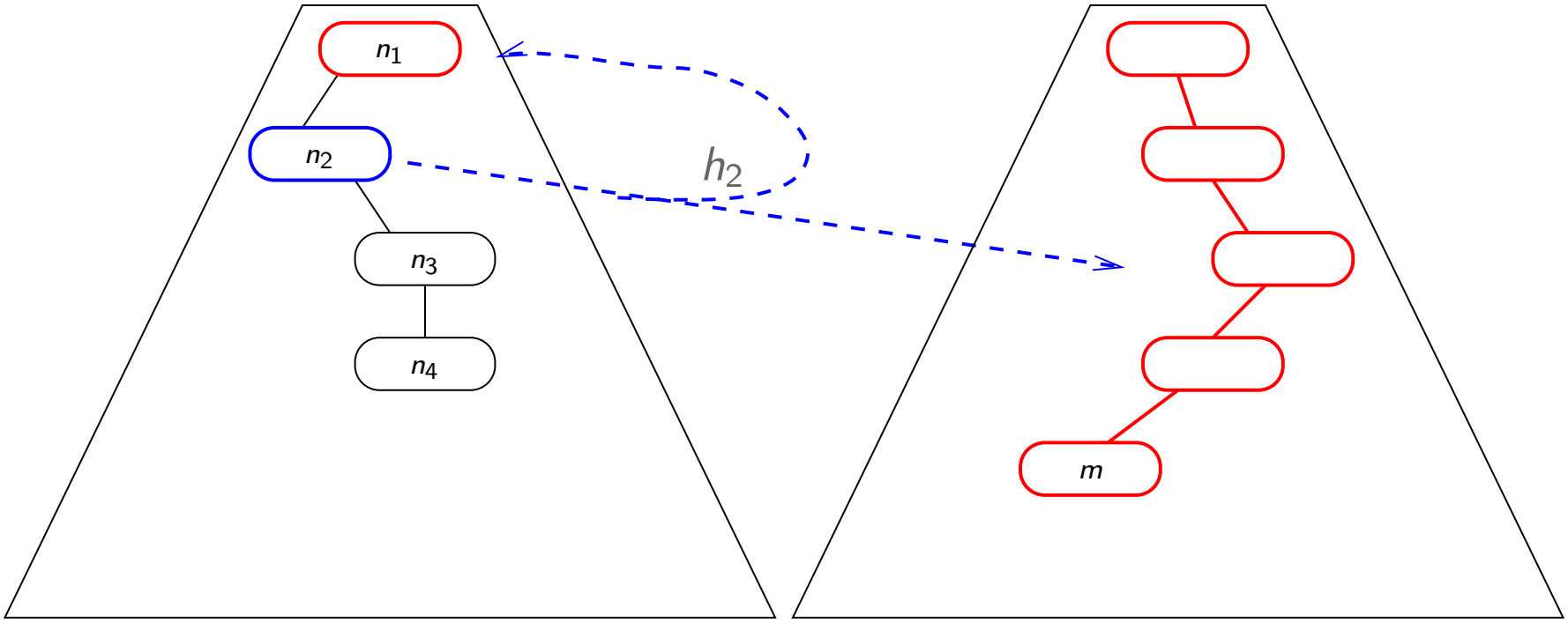

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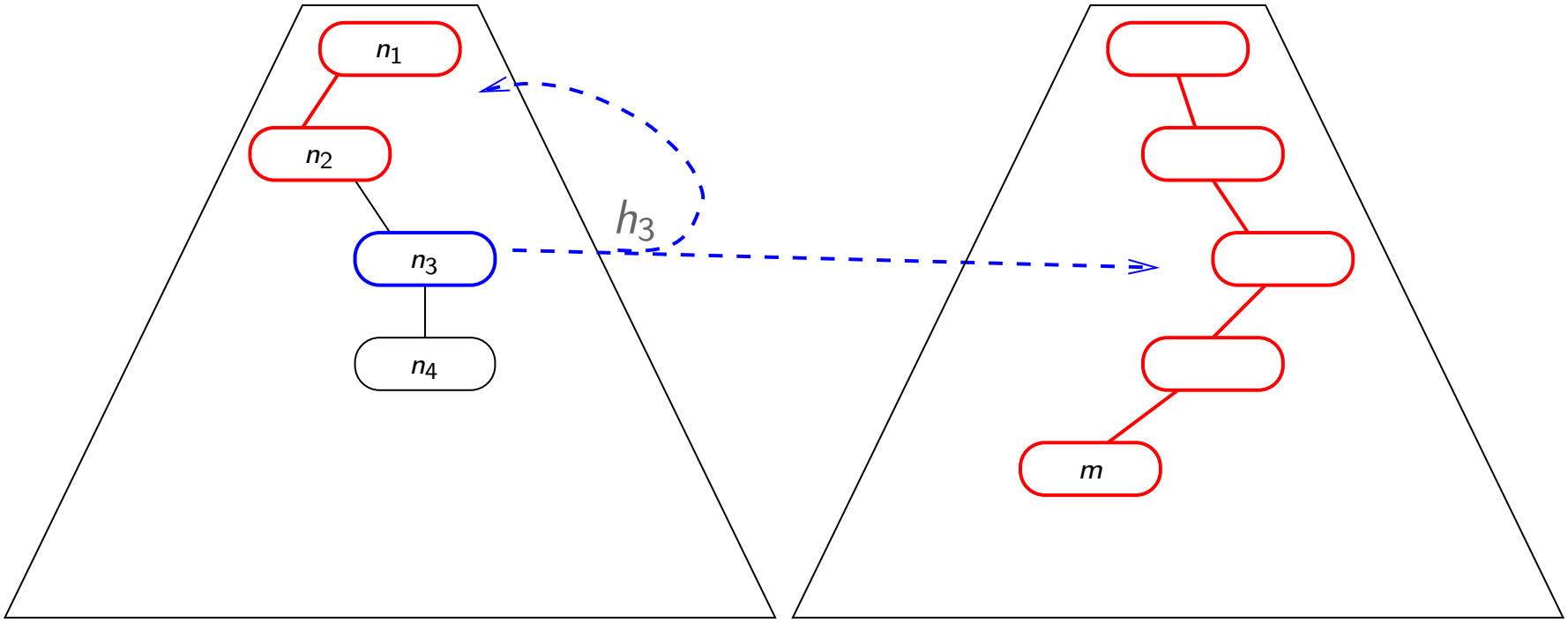

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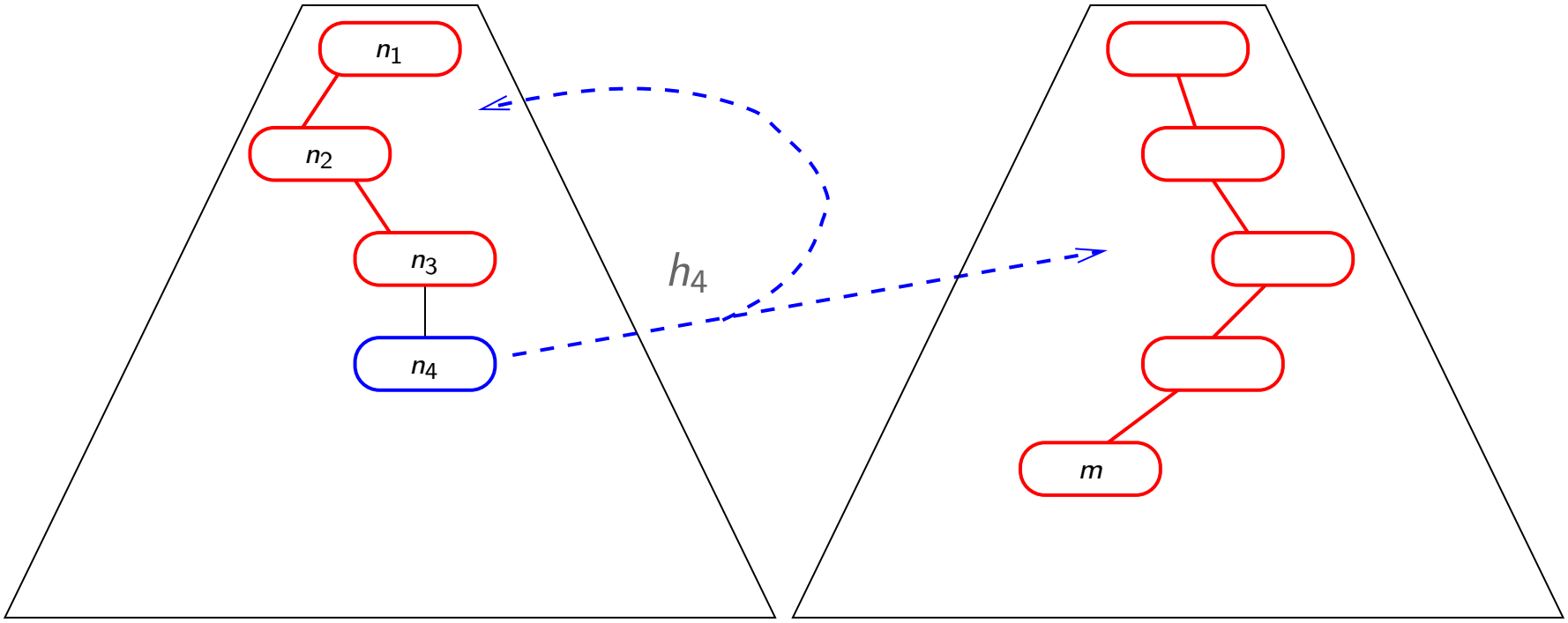

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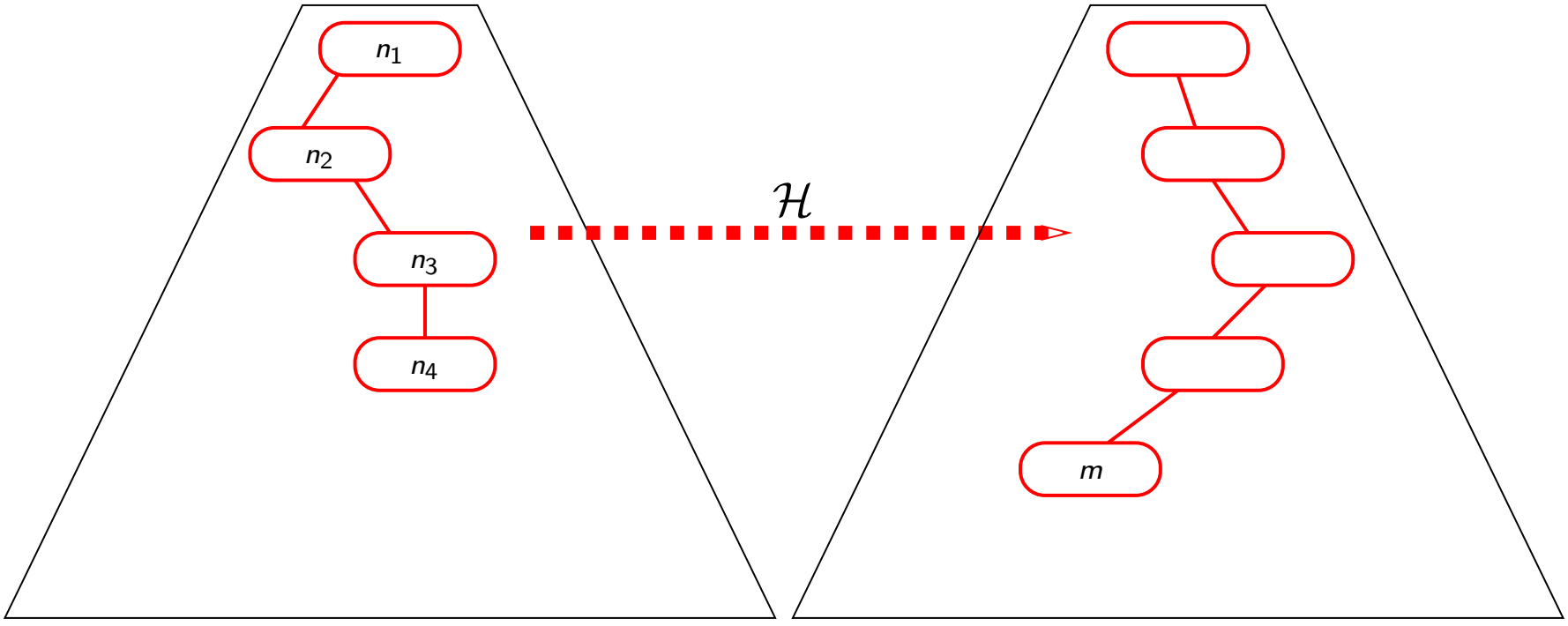

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$\mathcal{H} : \text{branch}(n_4) \rightarrow \text{branch}(m)$.

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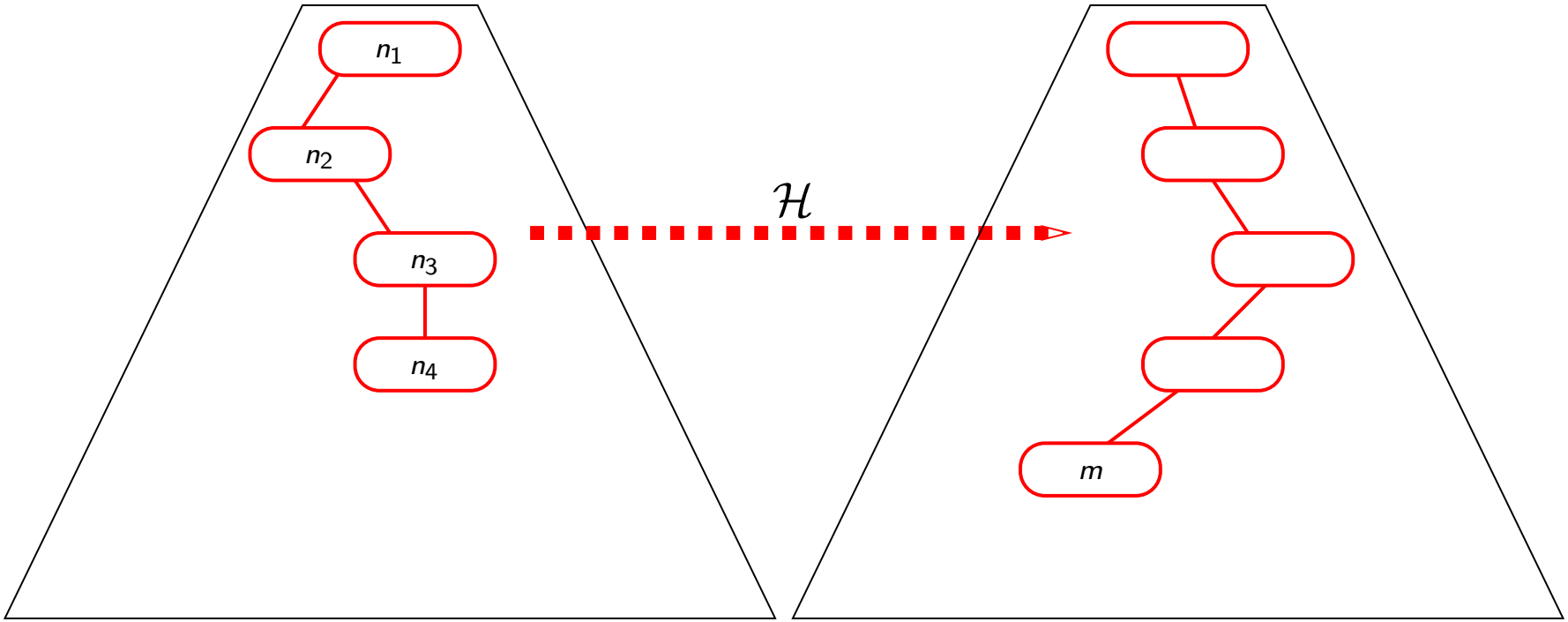

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Strong homomorphisms in both direction:

strongly homomorphically equivalent branches

Equivalence is NP-complete

Theorem

QWDPTs $\mathcal{T}_1$ and $\mathcal{T}_2$ in R3-normal form *are equivalent iff*

- *$\mathcal{T}_1$ and $\mathcal{T}_2$ have the same triple patterns, the same root, and*
- *if n_1 in $\mathcal{T}_1$ and n_2 in $\mathcal{T}_2$* introduce the same variable, *then* $\mathsf{branch}(n_1)$ *and* $\mathsf{branch}(n_2)$ *are strongly hom. equiv.*

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Checking equivalence of QWDPTs is NP-complete

Outline

CQ-tractable cases to tractable SPARQL

Each node of a QWDPT is a CQ $\Rightarrow$ natural question:

how do tractable fragments of CQ carry over?

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  $\#\cdot coNP$ in general

  $\#P$-hard even if every node is an acyclic CQ

Concluding remarks

Evaluation of SPARQL graph patterns via pattern trees

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Future work

- include more features (projection, union)
- more transformation rules
- tests and empirical results

Static Analysis and Optimization of Semantic Web Queries

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