

Exchanging More than Complete Data

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Father

Andy	Bob
Bob	Danny
Danny	Eddie

Mother

Carrie	Bob
--------	-----

Father(x, y) \rightarrow Parent(x, y)

Mother(x, y) \rightarrow Parent(x, y)

Parent(x, y) \wedge Parent(y, z) \rightarrow Gr-Parent(x, z)

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Source

Father(x, y) \rightarrow Pateras(x, y)
Gr-Parent(x, y) \rightarrow Pappoi(x, y)

Target

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$Pateras(x, y) \wedge Pateras(y, z) \rightarrow Pappoi(x, z)$

We can exchange more than complete data

- ▶ In data exchange we begin with a database instance
we begin with complete data
- ▶ What if we have a representation of a set of possible instances?
- ▶ We propose a new general formalism to exchange representations of possible instances
and apply it to incomplete instances and knowledge bases

Outline

Formalism for exchanging representations systems

Applications to incomplete instances

Applications to knowledge bases

What is the complexity of testing kb-solutions?

How can we compute a good kb-solution?

Concluding remarks

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Representation systems

A representation system is composed of:

- ▶ a set **W** of *representatives*
- ▶ a function *rep* from **W** to sets of instances

$$\mathcal{V} \xrightarrow{\text{rep}} \{l_1, l_2, l_3, \dots\} = \text{rep}(\mathcal{V})$$

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- ▶ Incomplete instances and knowledge bases are representation systems

In classical data exchange
we consider only *complete* data

$\mathcal{M} = (\mathbf{S}, \mathbf{T}, \Sigma)$ a schema mapping from \mathbf{S} to \mathbf{T}
 $I \in \text{Inst}(\mathbf{S}), J \in \text{Inst}(\mathbf{T})$

J is a *solution* for I under \mathcal{M} iff $(I, J) \models \Sigma$

$$J \in \text{Sol}_{\mathcal{M}}(I)$$

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We can extend this to set of instances:
given a set $\mathcal{X} \subseteq \text{Inst}(\mathbf{S})$

$$\text{Sol}_{\mathcal{M}}(\mathcal{X}) = \bigcup_{I \in \mathcal{X}} \text{Sol}_{\mathcal{M}}(I)$$

Extending the definition to representation systems

Given:

- ▶ a mapping $\mathcal{M} = (\mathbf{S}, \mathbf{T}, \Sigma)$
- ▶ a representation system $\mathcal{R} = (\mathbf{W}, \text{rep})$
- ▶ $\mathcal{U}, \mathcal{V} \in \mathbf{W}$

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Definition

\mathcal{U} is an \mathcal{R} -solution of \mathcal{V} if

$$\text{rep}(\mathcal{U}) \subseteq \text{Sol}_{\mathcal{M}}(\text{rep}(\mathcal{V}))$$

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or equivalently,

$$\forall J \in \text{rep}(\mathcal{U}), \exists I \in \text{rep}(\mathcal{V}) \text{ such that } J \in \text{Sol}_{\mathcal{M}}(I)$$

Universal solutions and strong representation systems

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Definition

$\mathcal{R} = (\mathbf{W}, \text{rep})$ is a *strong representation system* for \mathcal{C} if for every $\mathcal{M} \in \mathcal{C}$, and $\mathcal{V} \in \mathbf{W}$, there exists a $\mathcal{U} \in \mathbf{W}$ such that:

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Strong representation systems \Rightarrow universal solutions for representatives in \mathcal{R} can always be represented in the same system.

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Incomplete Instances

We consider:

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- ▶ Positive conditional instances: naive instances plus *conditions* for tuples
 - ▶ equalities between nulls, and between nulls and constants
 - ▶ conjunctions and disjunctions

$$\begin{array}{l|l} T(1, N_1) & true \\ R(1, N_1, N_2) & N_1 = N_2 \vee (N_1 = 2 \wedge N_2 = 3) \end{array}$$

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Semantics given by *valuations* of nulls (+ open world):

$$\text{rep}(\mathcal{I}) = \{K \mid \mu(\mathcal{I}) \subseteq K \text{ for some valuation } \mu\}$$

Strong representation systems for st-tgds

Let $\mathcal{M} = (\mathbf{S}, \mathbf{T}, \Sigma)$ be specified by source-to-target tgds
(st-tgd: $\phi_{\mathbf{S}}(\bar{x}) \rightarrow \exists \bar{y} \psi_{\mathbf{T}}(\bar{x}, \bar{y})$, $\phi_{\mathbf{S}}$ and $\psi_{\mathbf{T}}$ are CQs)

(FKMP03)

For every complete instance I there exists a naive instance \mathcal{J} s.t.

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Proposition

Naive instances are not a strong representation system for st-tgds

Idea

$\text{Mng}(N, \text{Alice})$	$\text{Mng}(x, y) \rightarrow \text{Reports}(x, y)$
	$\text{Mng}(x, x) \rightarrow \text{Self-Mng}(x)$

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Mng(N , Alice)	Mng(x, y)	\rightarrow	Reports(x, y)	?
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$\text{Mng}(N, \text{Alice})$	$\text{Mng}(x, y) \rightarrow \text{Reports}(x, y)$	$\text{Reports}(N, \text{Alice})$	true $N = \text{Alice}$
	$\text{Mng}(x, x) \rightarrow \text{Self-Mng}(x)$	$\text{Self-Mng}(\text{Alice})$	

Positive conditional instances are enough

Theorem

Positive conditional instances are a strong representation system for st-tgds.

Mng(N , Alice)	Mng(x , y)	→	Reports(x , y)	Reports(N , Alice)	$true$ $N = \text{Alice}$
	Mng(x , x)	→	Self-Mng(x)	Self-Mng(Alice)	

Corollary

Positive conditional instances are a strong representation system for SO-tgds.

Positive conditional instances are *exactly* the needed representation system

Theorem

Positive conditional instances are minimal:

- ▶ *equalities between nulls*
- ▶ *equalities between constant and nulls*
- ▶ *conjunctions and disjunctions*

are all needed for a strong representation system of st-tgds

Positive conditional instances match the complexity bounds of classical data exchange

Theorem

For positive conditional instances and (fixed) st-tgds

- ▶ *Universal solutions can be constructed in polynomial time.*
- ▶ *Certain answers of unions of conjunctive queries can be computed in polynomial time.*
- ▶ *Certain answers of unions of conjunctive queries with one inequality per disjunct can be computed in polynomial time.*

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The semantics of *knowledge bases* is given by sets of instances

Knowledge base over \mathbf{S} : (I, Γ) s.t.

$$I \in \text{Inst}(\mathbf{S})$$

Γ a set of rules over \mathbf{S}

Semantics: finite models

$$\text{Mod}(I, \Gamma) = \{K \in \text{Inst}(\mathbf{S}) \mid I \subseteq K \text{ and } K \models \Gamma\}$$

We can apply our formalism
to knowledge bases

(I_2, Γ_2) is a *kb-solution* for (I_1, Γ_1) under \mathcal{M} iff

$$\text{Mod}(I_2, \Gamma_2) \subseteq \text{Sol}_{\mathcal{M}}(\text{Mod}(I_1, \Gamma_1))$$

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We study the following decision problem

CHECK-KB-SOL:

Input: $\mathcal{M} = (\mathbf{S}, \mathbf{T}, \Sigma)$ with Σ st-tgds
 (I_1, Γ_1) knowledge base over \mathbf{S} with Γ_1 tgds
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Output: Is (I_2, Γ_2) a kb-solution of (I_1, Γ_1) under \mathcal{M} ?

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Undecidability is a consequence of using \exists in knowledge bases

CHECK-FULL-KB-SOL: Γ_1, Γ_2 full-tgds (no \exists)

The complexity of CHECK-FULL-KB-SOL

Theorem

CHECK-FULL-KB-SOL is *EXPTIME*-complete.

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CHECK-FULL-KB-SOL is *EXPTIME*-complete.

Theorem

If $\mathcal{M} = (\mathbf{S}, \mathbf{T}, \Sigma)$ is fixed

CHECK-FULL-KB-SOL is $\Delta_2^P[O(\log n)]$ -complete.

$\Delta_2^P[O(\log n)] \equiv P^{NP}$ with only a *logarithmic*
number of calls to the *NP* oracle.

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What are good knowledge-base solutions?

- ▶ first choice: universal kb-solutions, but
- ▶ there are some other kb-solutions desirable to materialize

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(I_2, Γ_2) is a *minimal kb-solution* of (I_1, Γ_1) under \mathcal{M} if

$$\text{Mod}(I_2, \Gamma_2) \equiv_{\min} \text{Sol}_{\mathcal{M}}(\text{Mod}(I_1, \Gamma_1))$$

We consider minimal kb-solutions as the good solutions

Two requirements to construct minimal knowledge-base solutions

Given (I_1, Γ_1) and Σ , when constructing a minimal-knowledge base solution (I_2, Γ_2) we would want:

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Γ_2 is *optimum-safe* if for every other safe set Γ'

$$\Gamma_2 \models \Gamma'$$

Safe sets: data independent target knowledge bases

Unfortunately, optimum-safe sets are not (always) FO-expressible

Theorem

There exist sets Γ_1 (full & acyclic) and Σ (full) s.t.

no FO sentence is optimum-safe for Γ_1 and Σ

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In the paper: an algorithm to compute *good* safe sets

- ▶ works for full & acyclic Γ_1 and full Σ
- ▶ we use, unfolding, rewriting, and composition of mappings

In the paper: a complete strategy for computing minimal knowledge-base solutions

Given Γ_1 and Σ :

- ▶ We compute Γ_2 safe for Γ_1 and Σ
- ▶ We then compute minimal set Γ^* (implied by Γ_1) such that
for every I_1

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is a minimal kb-solution for (I_1, Γ_1) .

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Next step: apply our general setting to the Semantic Web

- ▶ Semantic Web data have *nulls* (blank nodes)
- ▶ Semantic Web specifications have rules (RDFS, OWL)

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