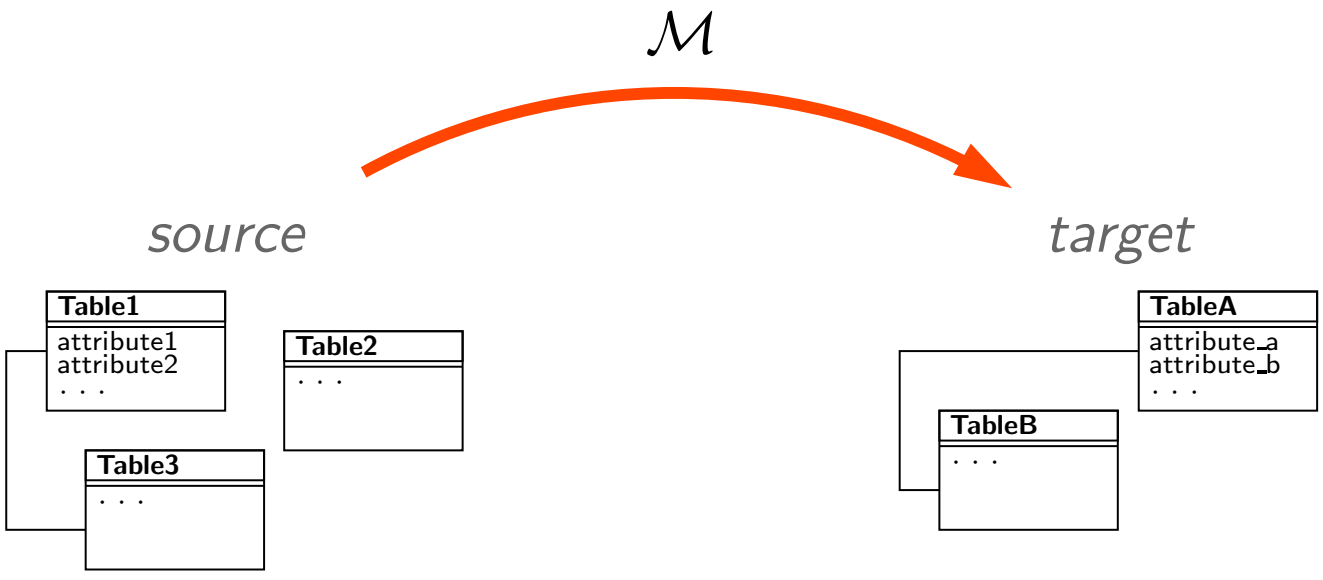
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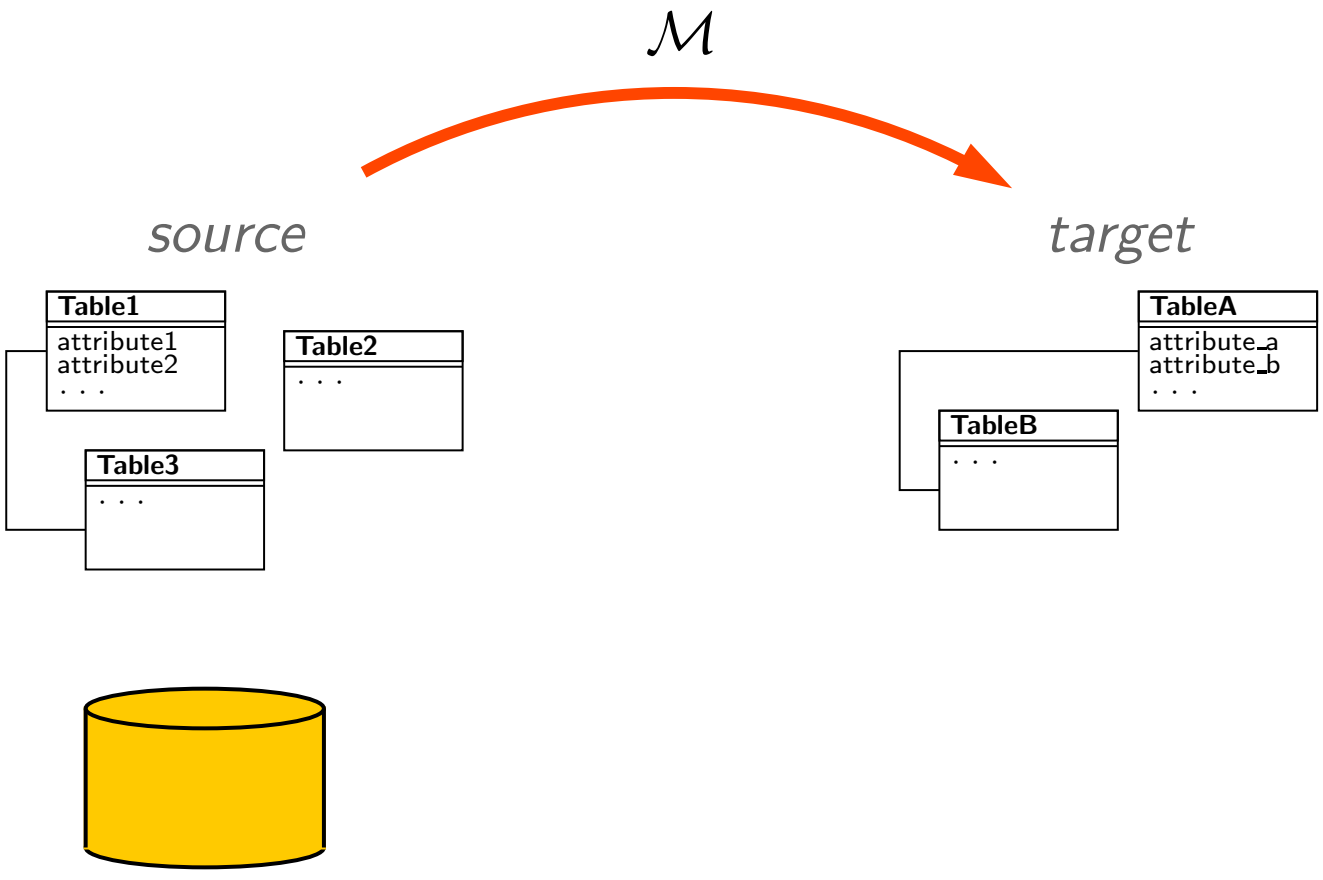
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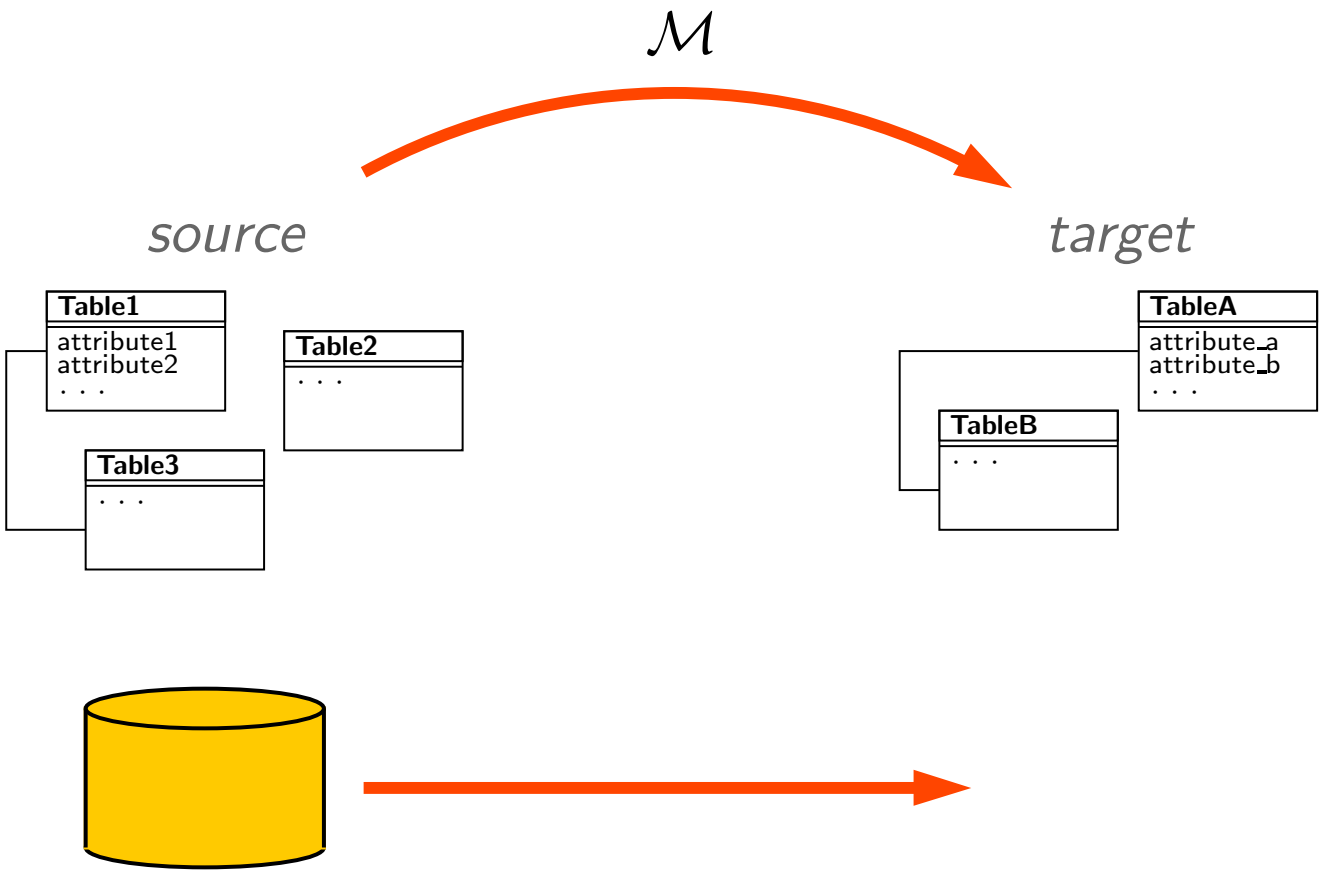
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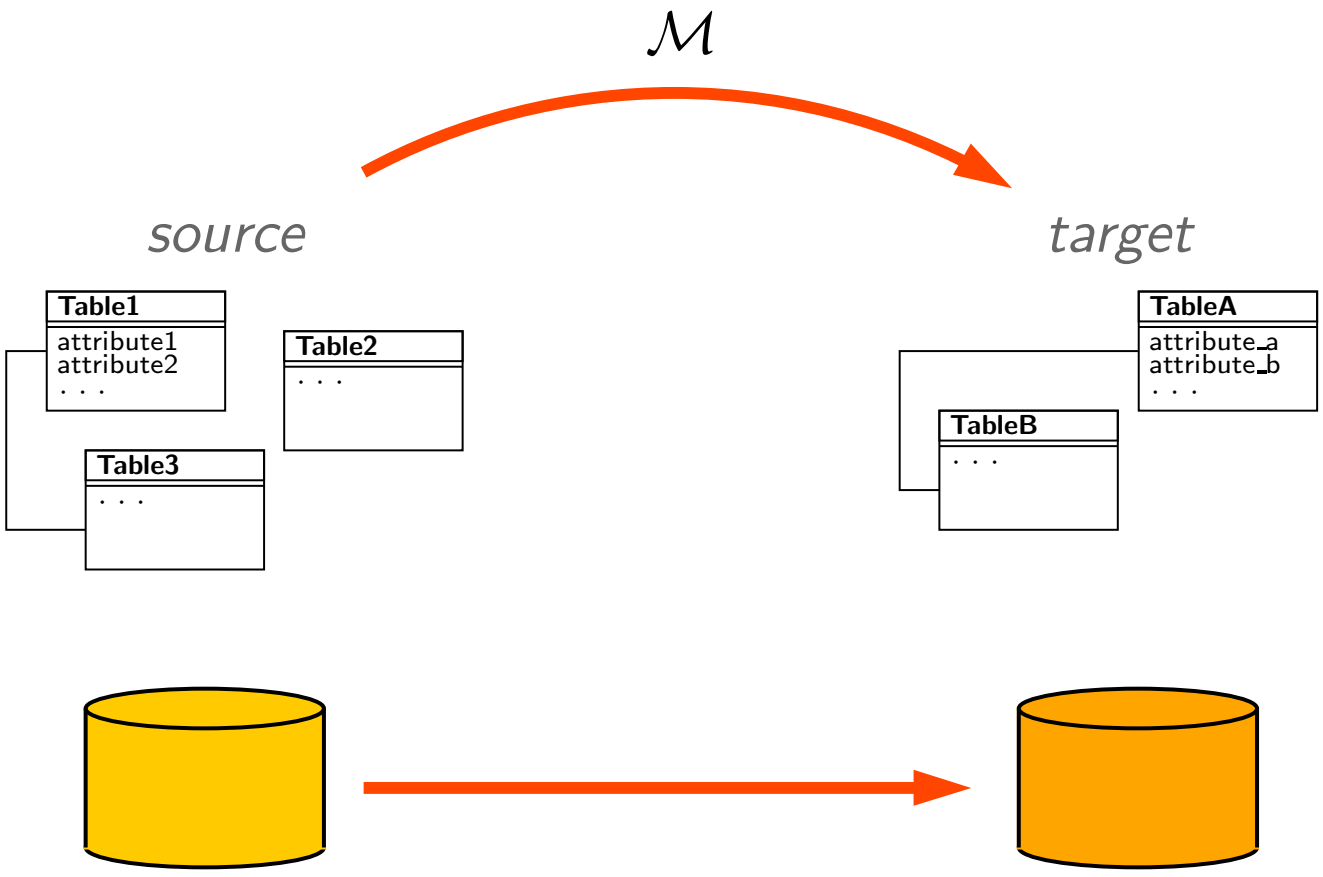
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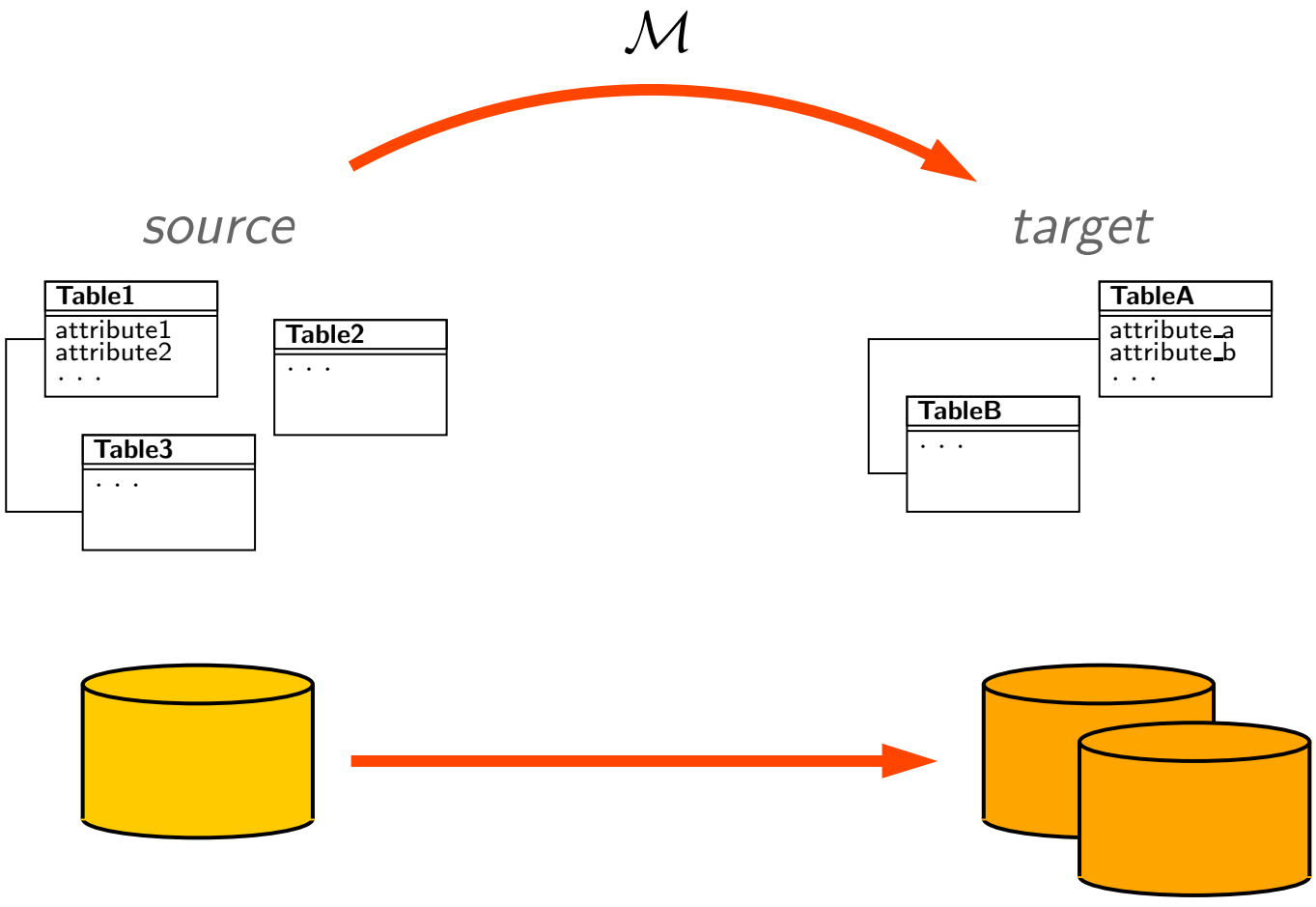
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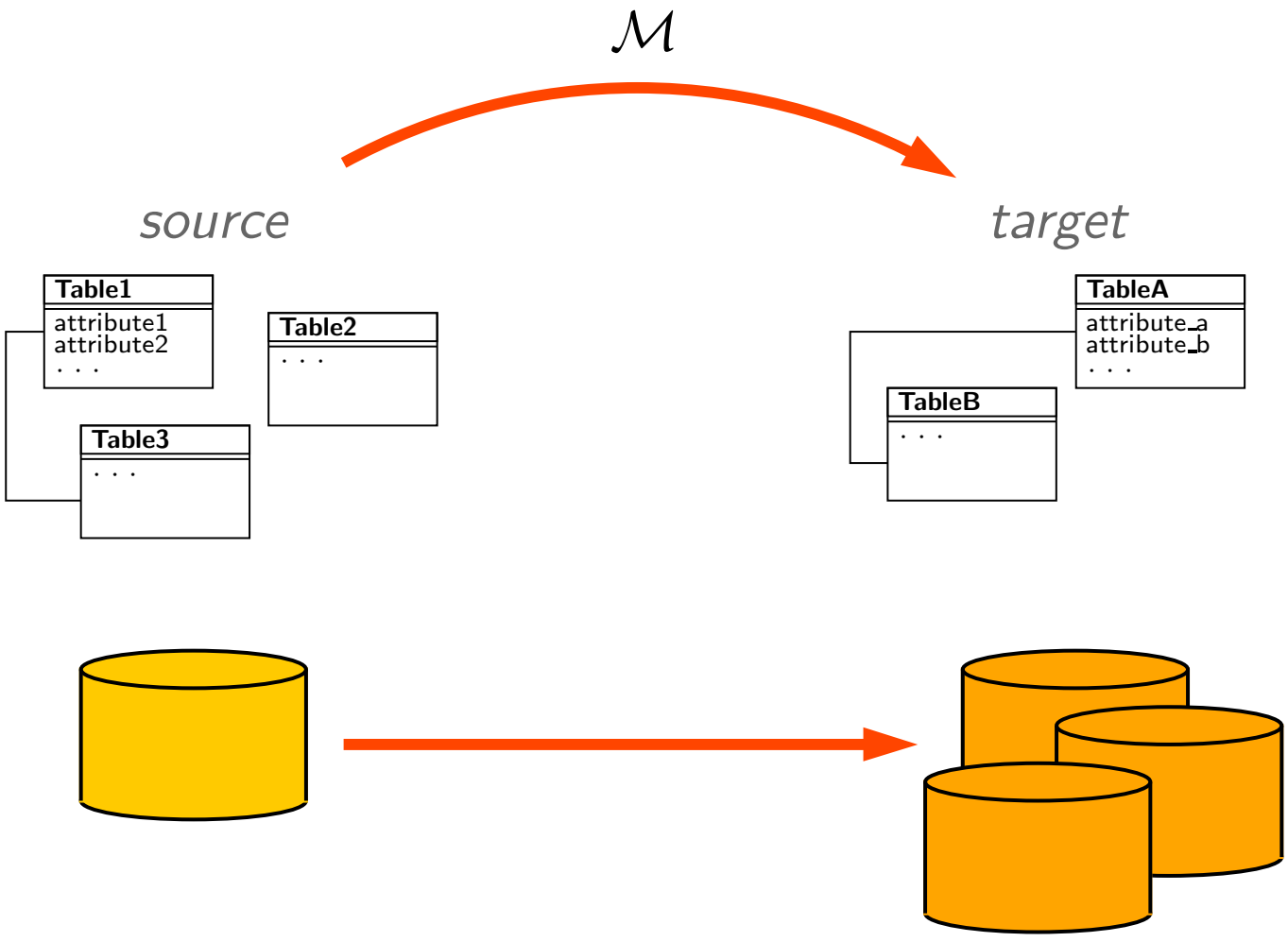
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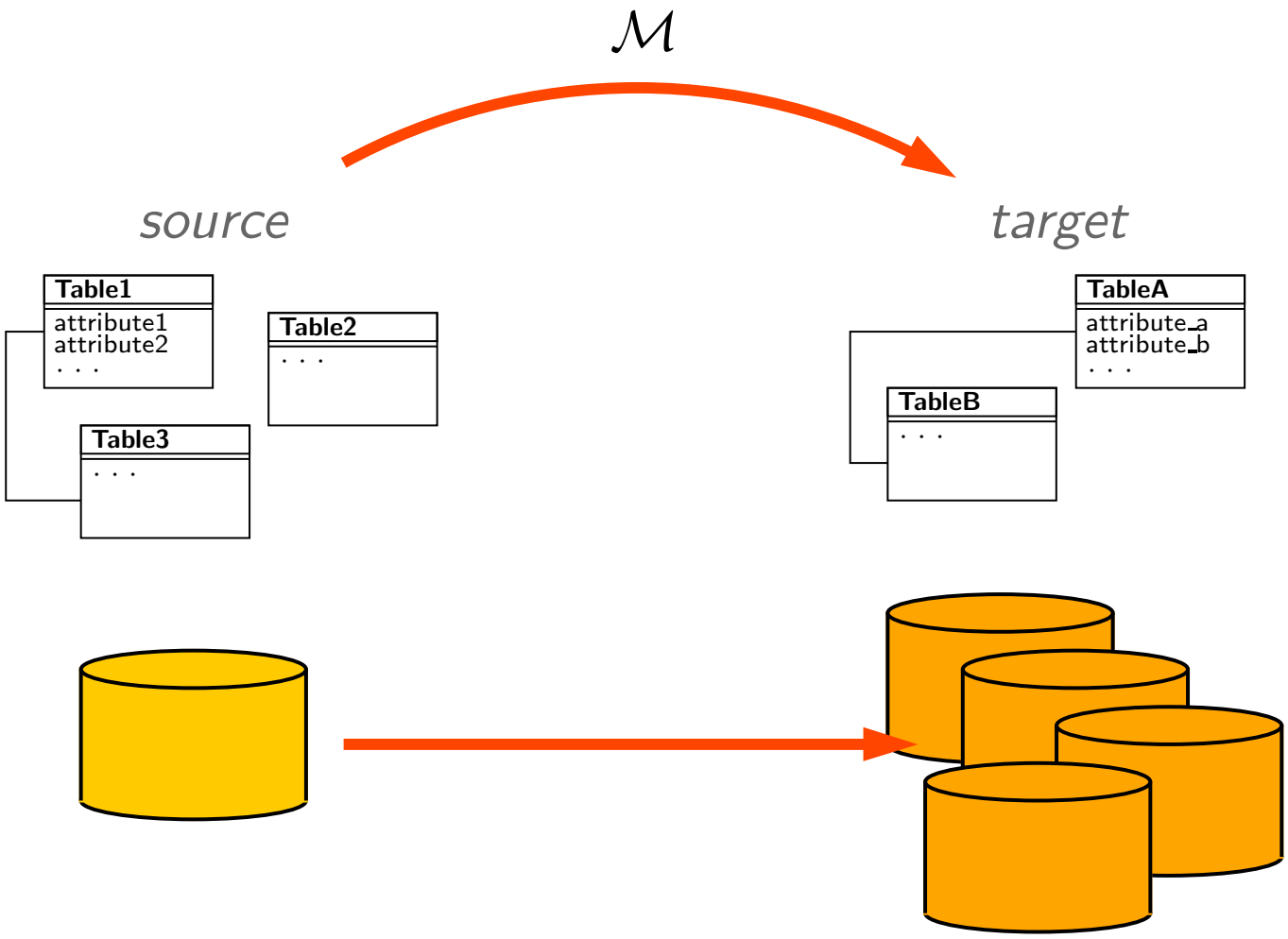
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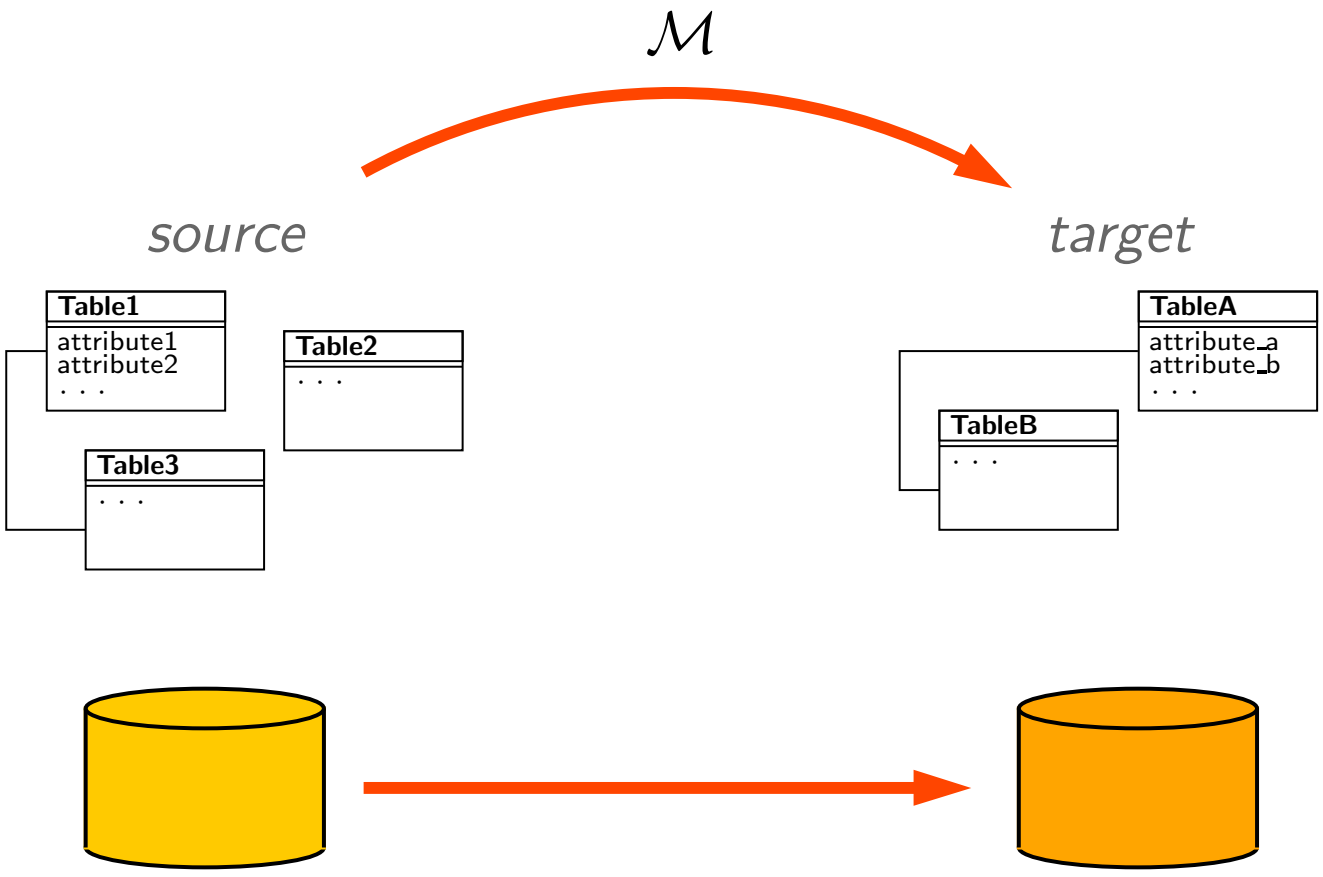
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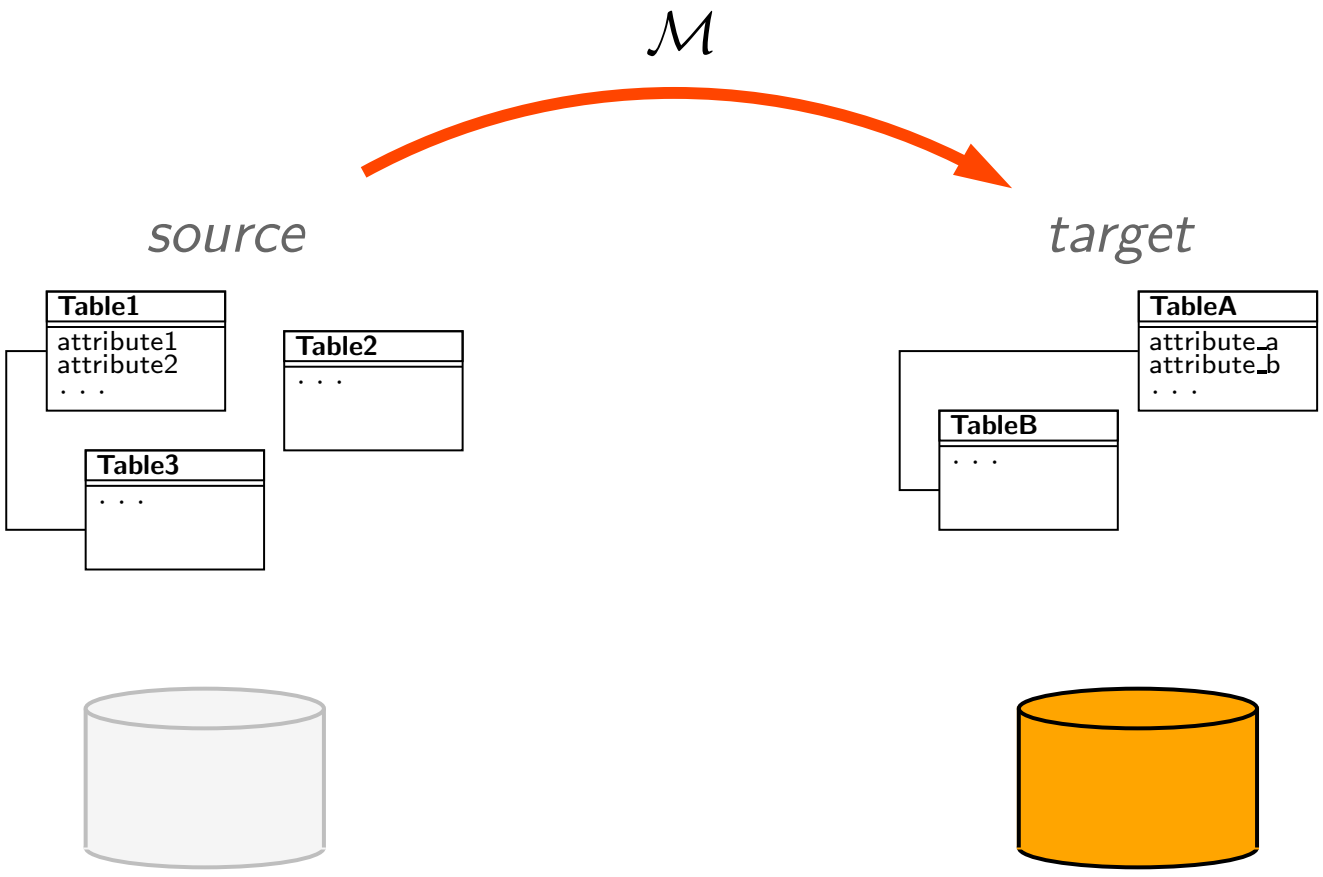
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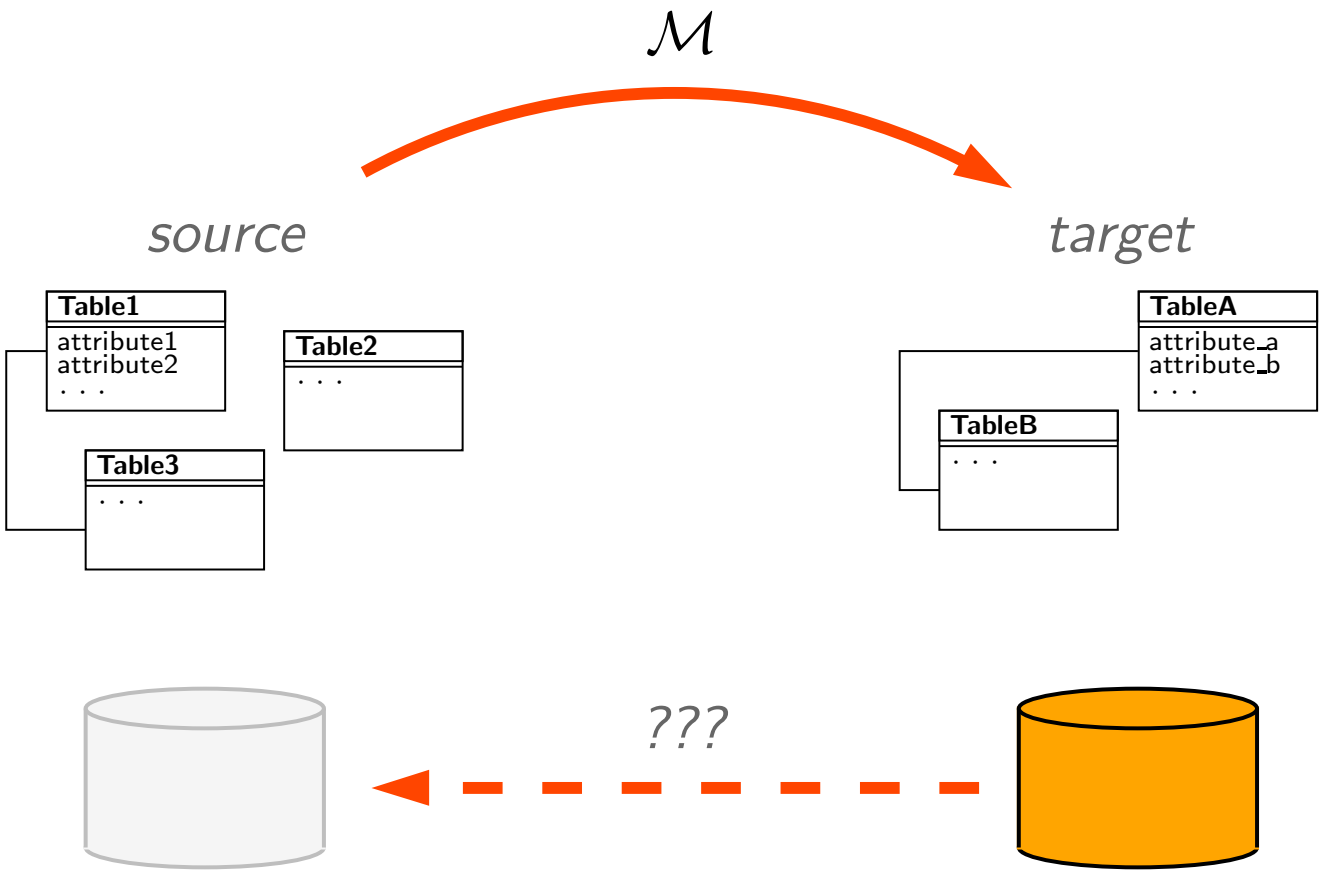
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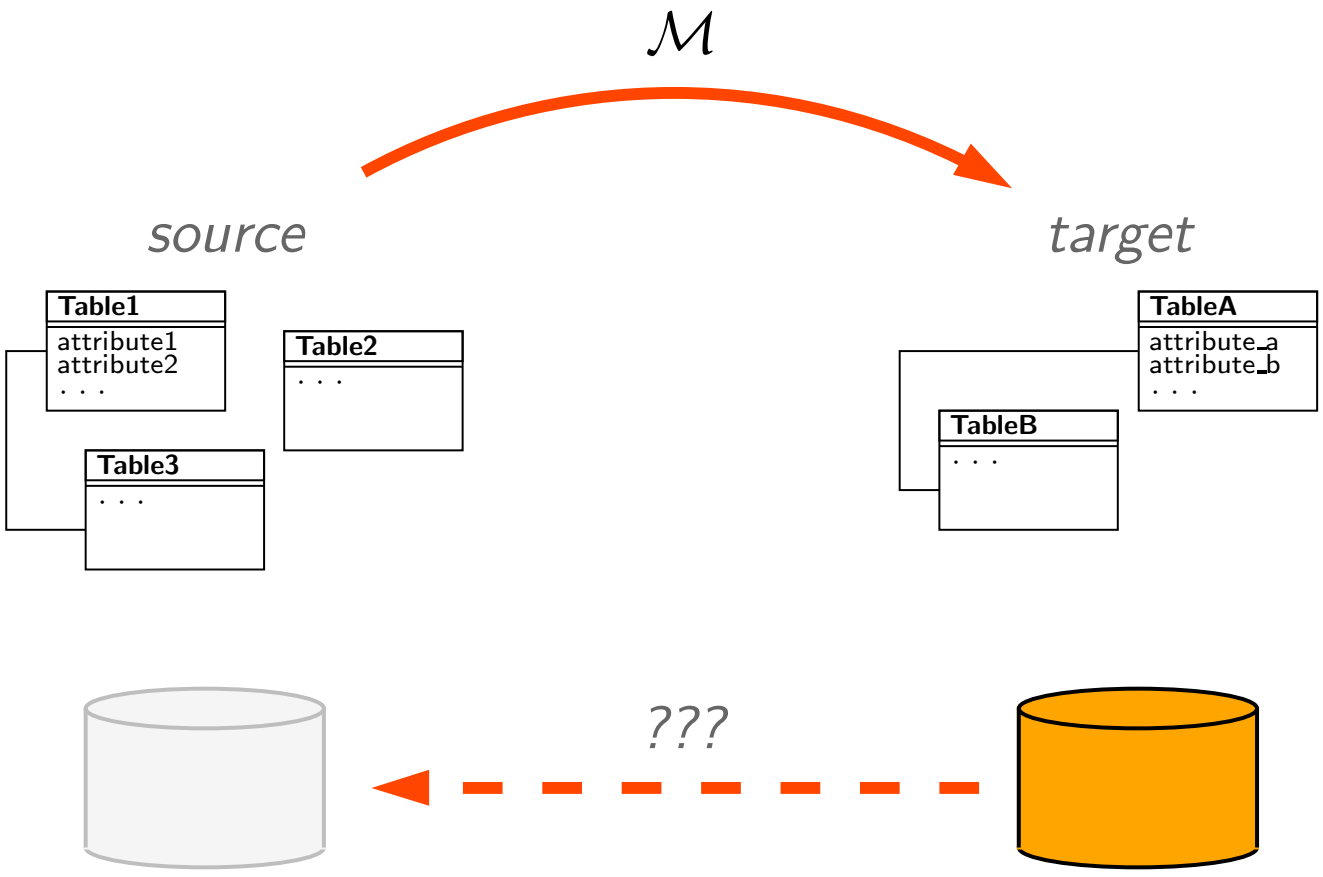
How do we *recover* exchanged data?



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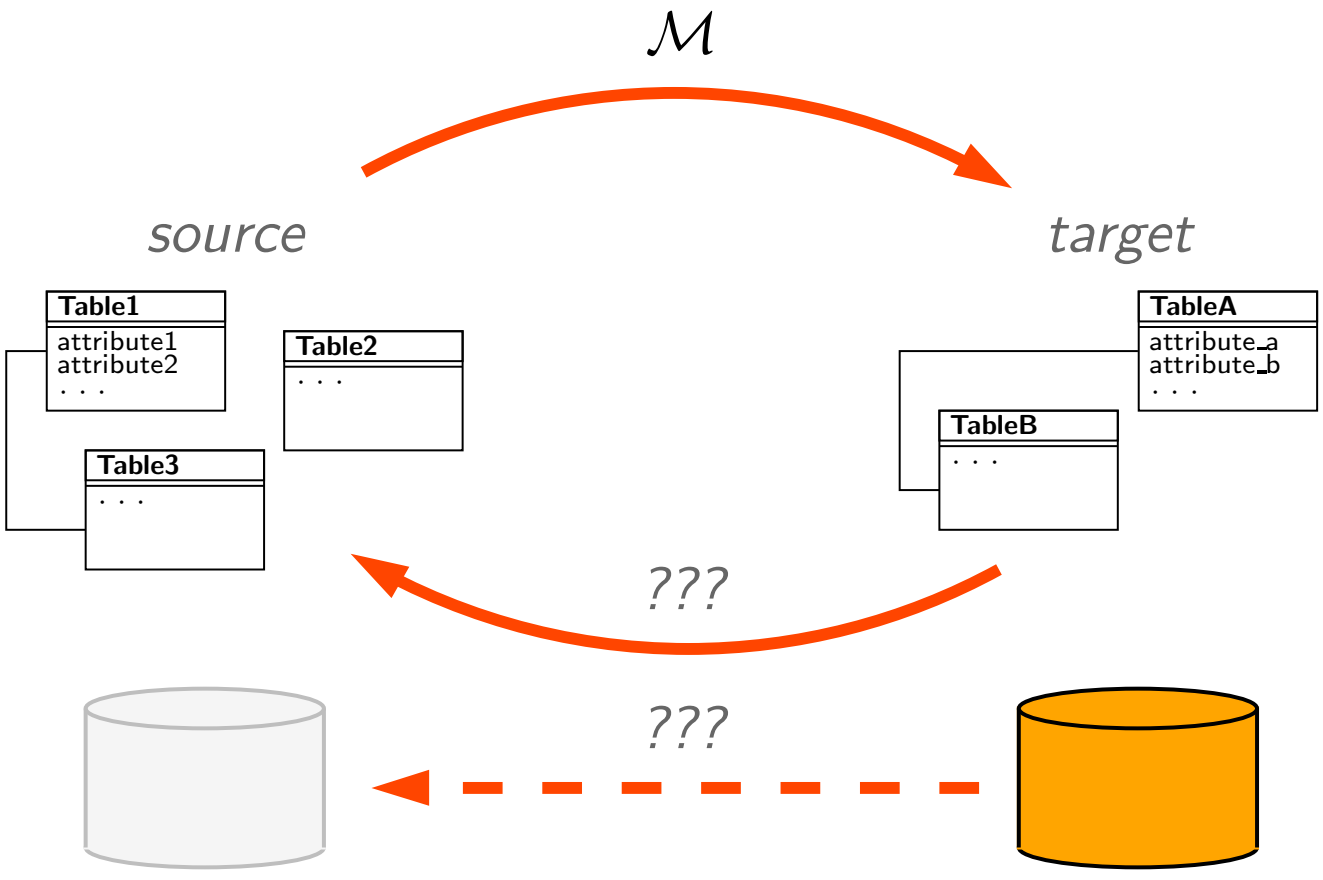

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Question

Is there an alternative way
to recover (at least part of) the exchanged data?

The Recovery of a Schema Mapping: Bringing Exchanged Data Back

Marcelo Arenas Jorge Pérez Cristian Riveros

Departamento de Ciencia de la Computación
Pontificia Universidad Católica de Chile

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$\mathcal{M}'$ is a maximum recovery of $\mathcal{M}$.

Our contributions

Conceptually:

- recovery and maximum recovery

Technically:

- characterization of the existence of a maximum recovery
- a positive result for the most common mappings
- comparison with (quasi-)inverse
- algorithms
- complexity study

Outline

A bit of notation...

A *mapping* $\mathcal{M}$ is a set of pairs (I, J) with

- I a source instance,
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If $(I, J) \in \mathcal{M}$ then J *is a solution for* I under $\mathcal{M}$

- $J \in \mathrm{Sol}_{\mathcal{M}}(I)$.

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$\mathcal{M}'$ is a *recovery* of $\mathcal{M}$ if for every source instance I

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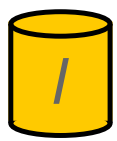

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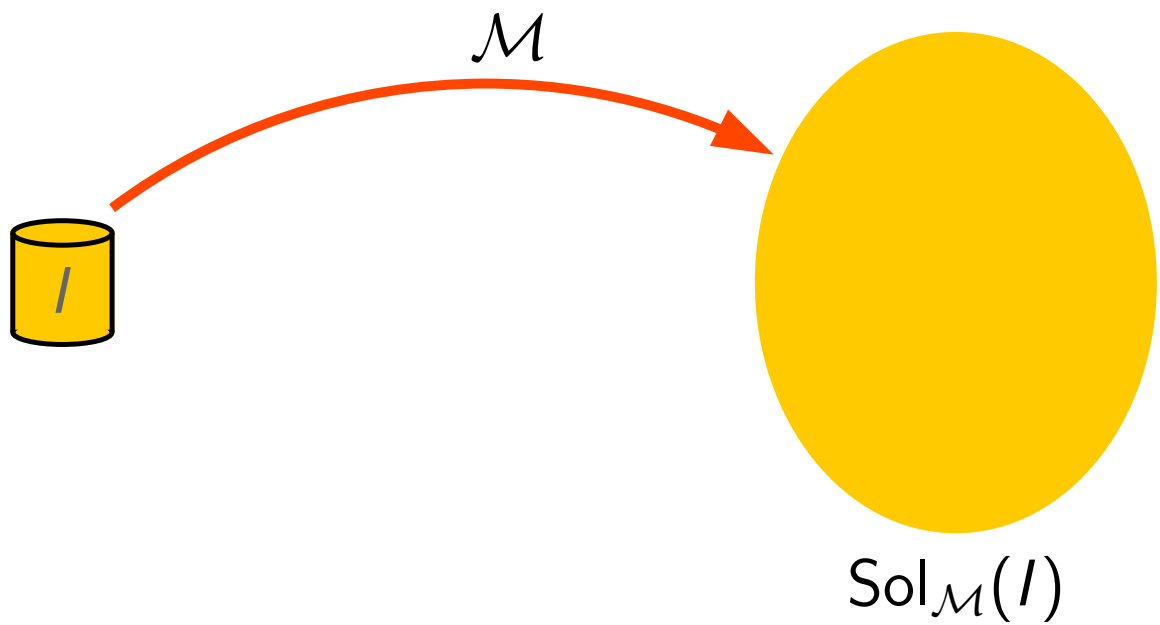

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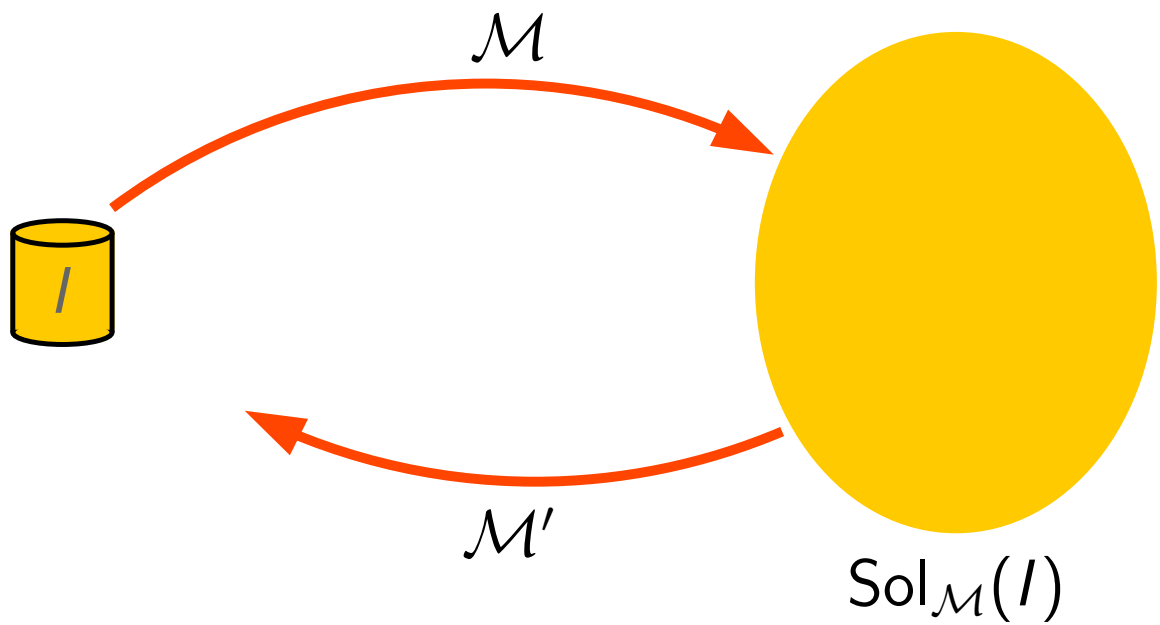

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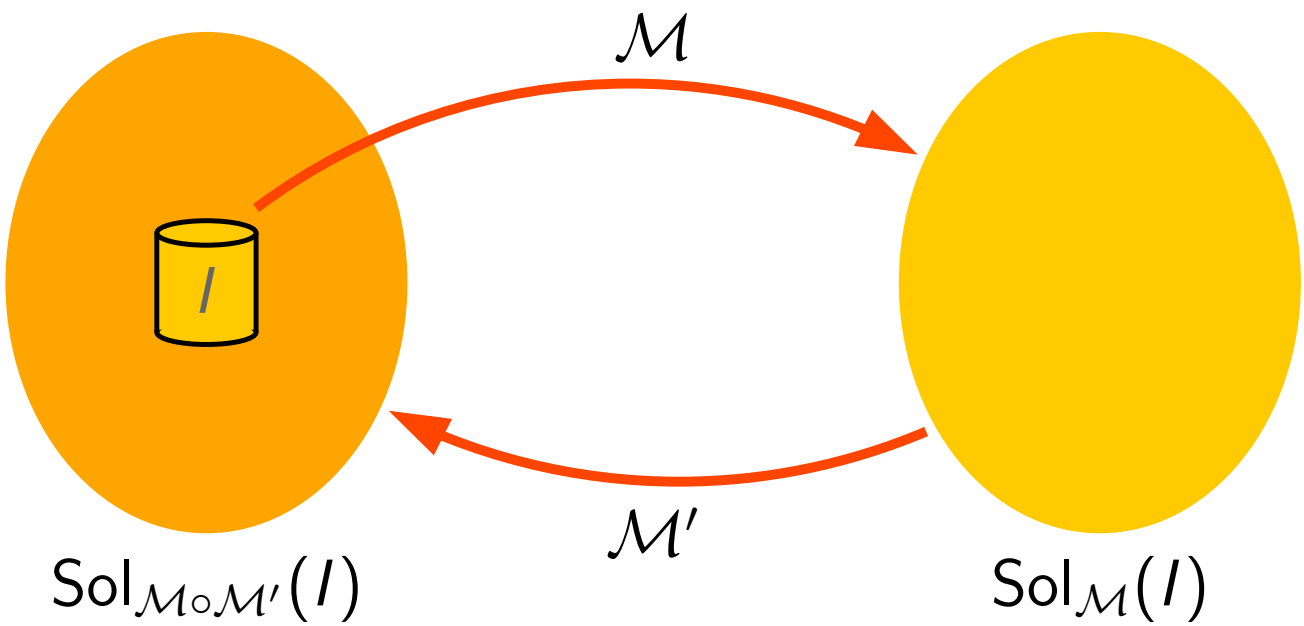

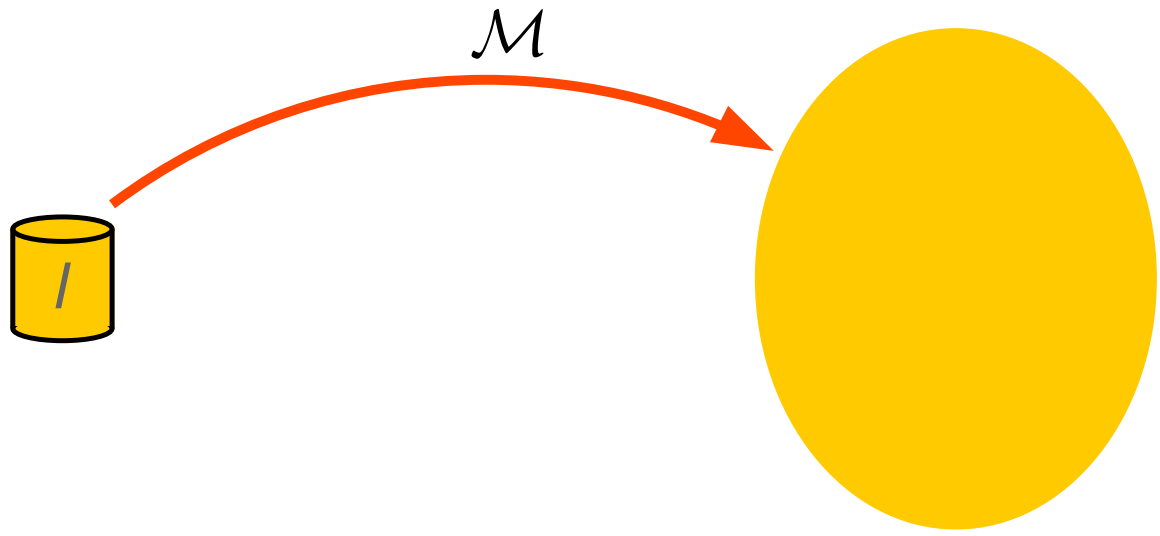
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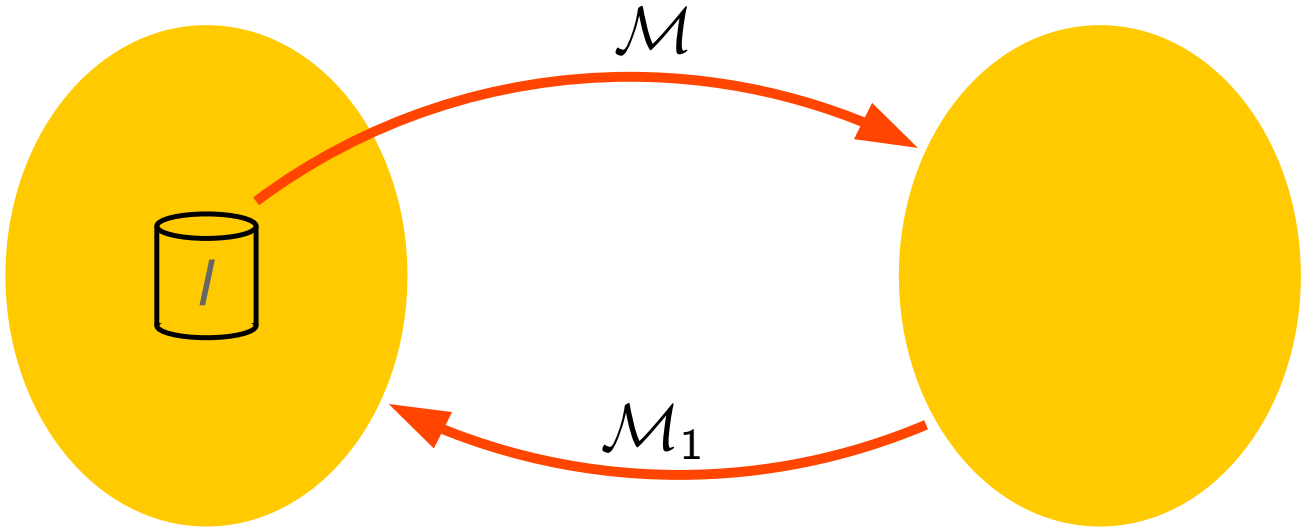
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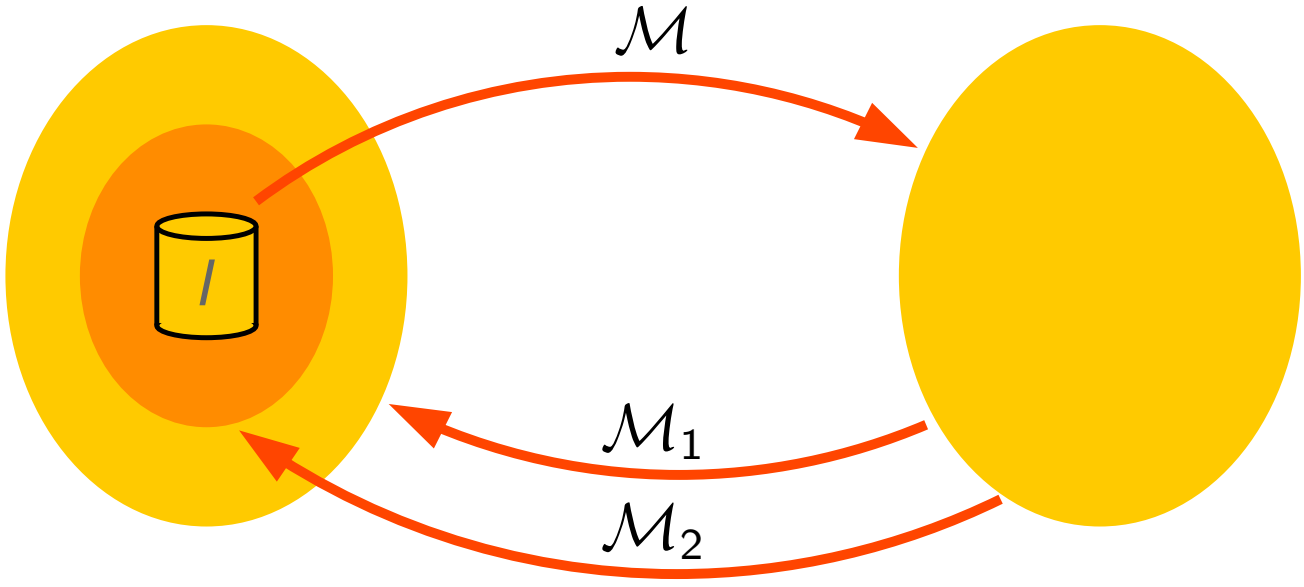
$\mathcal{M}_1$


$\mathcal{M}$
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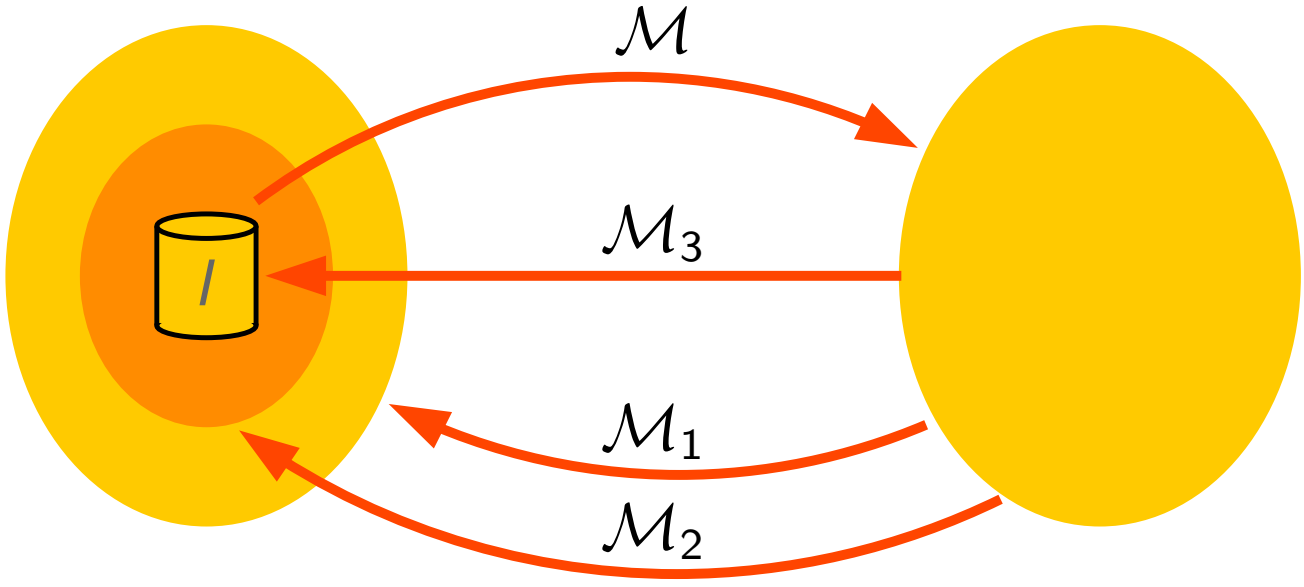
$\mathcal{M}$
$\mathcal{M}_3$
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$\mathcal{M}_1$
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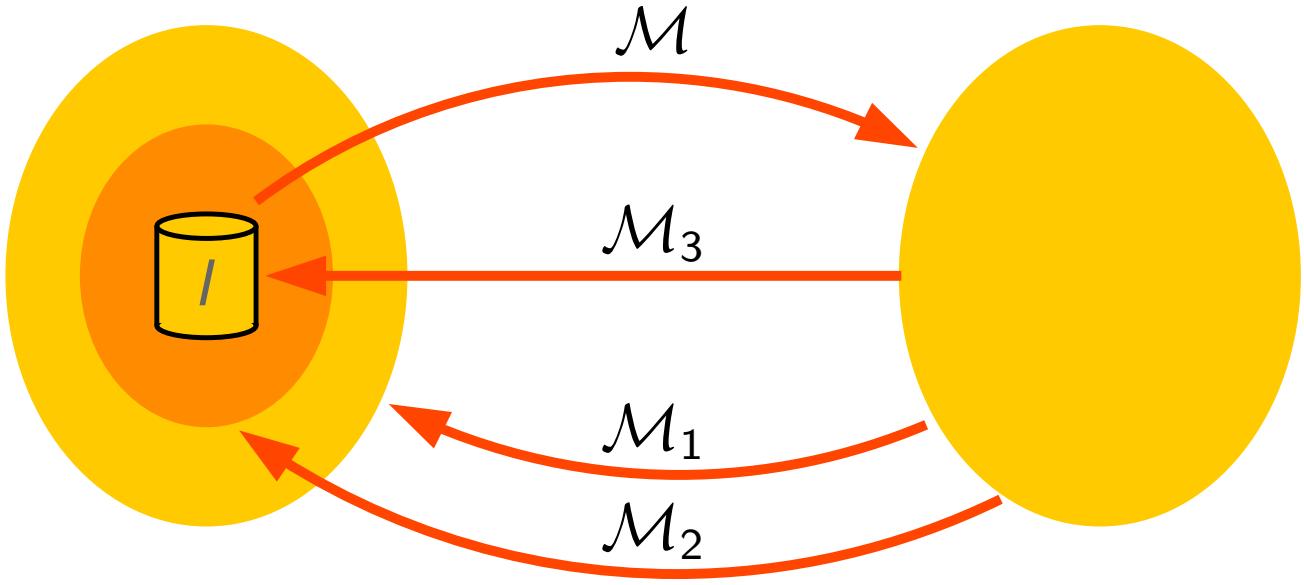
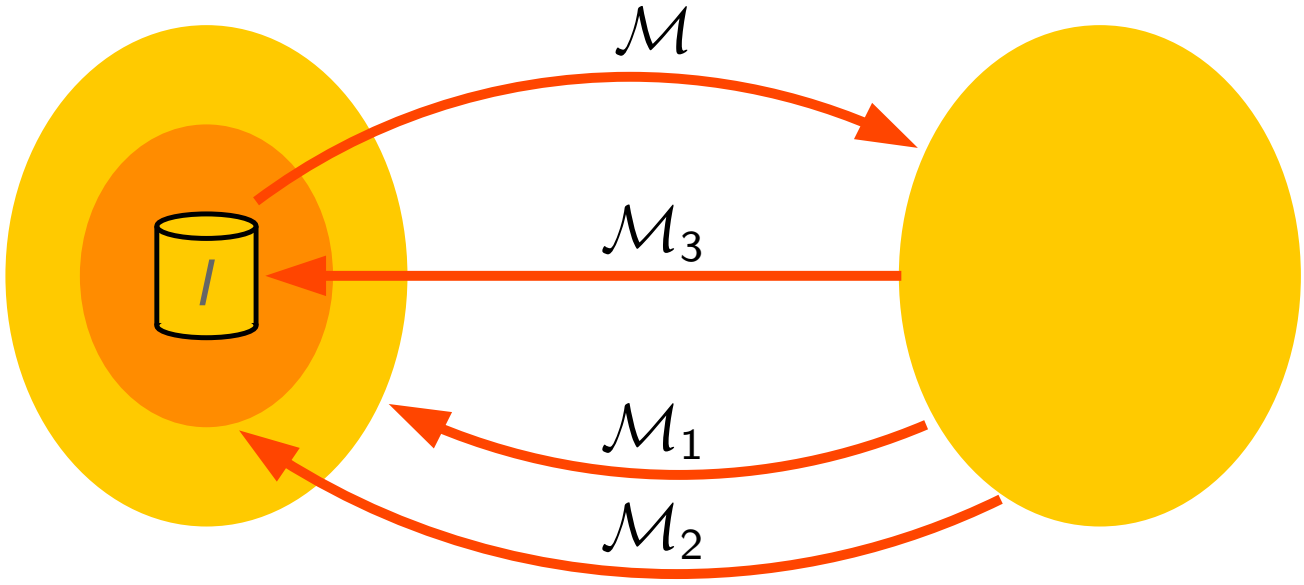


$\mathcal{M}'$ is *at least as informative as* $\mathcal{M}''$ for $\mathcal{M}$



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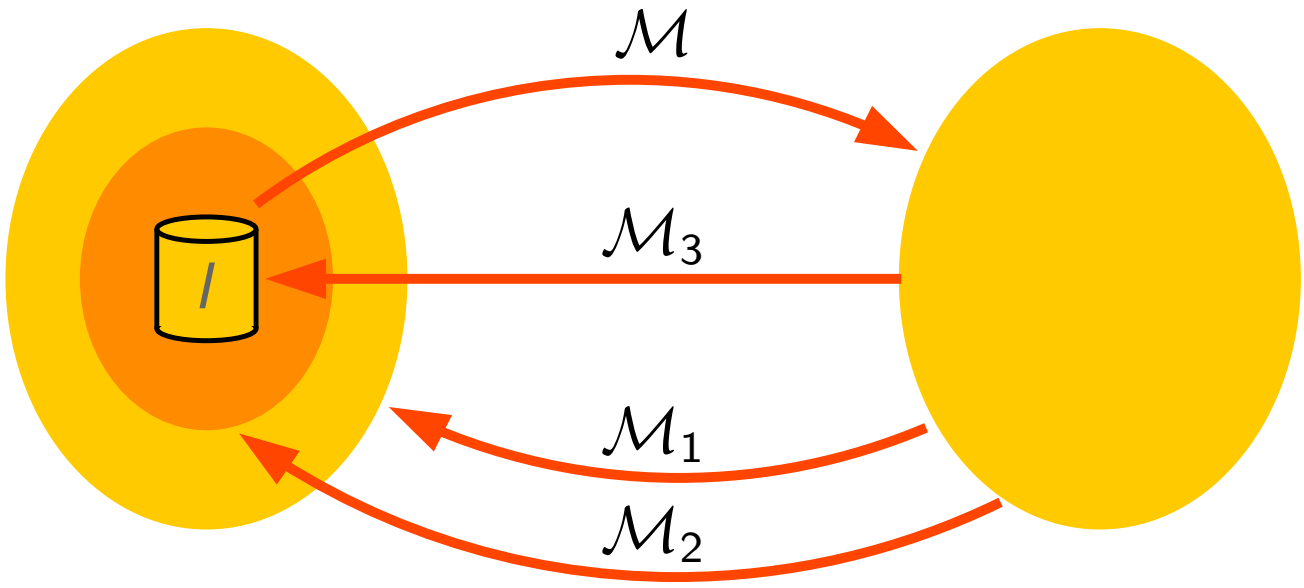

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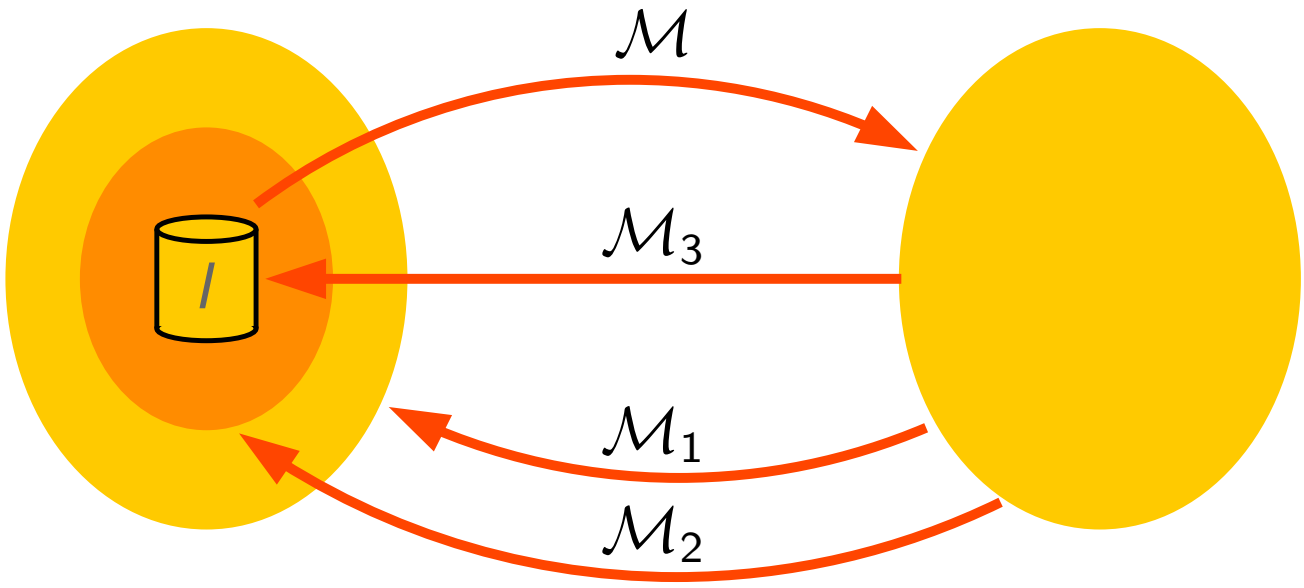

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Definition

$\mathcal{M}'$ is a *maximum recovery* of $\mathcal{M}$ iff
$\mathcal{M}'$ is a maximum w.r.t. $\preceq$.

A necessary and sufficient condition for the existence of maximum recoveries

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J is a *witness solution* for I under $\mathcal{M}$ if for every other instance I',

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Theorem

$\mathcal{M}$ has a maximum recovery iff every source instance has a witness solution.

FO-to-CQ dependencies and specification of mappings

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$$\varphi_{\mathbf{S}}(\bar{x}) \rightarrow \psi_{\mathbf{T}}(\bar{x})$$

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- A set Σ of dependencies *specifies* $\mathcal{M}$:

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Theorem

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$$\mathcal{M}: \quad P(x,y) \wedge P(y,z) \quad \rightarrow \quad R(x,z) \wedge T(y)$$

- Has neither an inverse nor a quasi-inverse [FKPT07].

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Maximum recoveries
are not guaranteed to exist in general.

Proposition

There are mappings specified by

- **CQ-to-CQ$^{\neq}$**
- **CQ-to-UCQ**
- **CQ-to-CQ$^{\neg}$**

dependencies, that have no maximum recovery.

Maximum recoveries strictly generalize [Fag06]-inverses

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In the paper: similar results for quasi-inverses.

Outline

Algorithm idea: *explicit the relationships* in the mapping, then *reverse the arrows*.

Example (full **FO-to-CQ**)

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Expliciting in the non-full case is more complicated.

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2. Reverse the arrows forcing *constants* in the source (**Const**$(\cdot)$).

Algorithm: quadratic in the full case, exponential in general

	full **FO-to-CQ**	**FO-to-CQ**
Output:		
Size:		
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Some *highlights*:

- Obtains small inverses and quasi-inverses in the full case (exponential in [FKPT07]).

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In the paper: a formal justification of $\mathbf{CQ^{Const(\cdot)}}$**-to-FO** as the language needed to specify maximum recoveries.

Complexity results

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Input: $\mathcal{M}$ *and* $\mathcal{M}'$ **CQ-to-CQ** *dependencies*
Question: *Is* $\mathcal{M}'$ *a* recovery *of* $\mathcal{M}$*?*

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$$\begin{array}{rl} \textit{coNP-complete} & \textit{if full } \mathcal{M} \textit{ and } \mathcal{M}' \\ \Pi_2^P\textit{-complete} & \textit{if full } \mathcal{M} \end{array}$$

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Also undecidable *for inverses and quasi-inverses.*

Outline

Maximum recovery: alternative notion on how to *reverse* schema mappings.

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- simple, following a *best effort* approach
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Technically:

- characterization of the existence of maximum recoveries
- positive result for the most common type of mappings
- algorithm for a large class of mappings
- complexity results
- new and general results for inverses and quasi-inverses

The Recovery of a Schema Mapping: Bringing Exchanged Data Back

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Pontificia Universidad Católica de Chile

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