Semantics and Complexity of SPARQL

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A simple RDF query language

```
SELECT ?Name ?Email
WHERE
{
  ?X :name ?Name
  ?X :email ?Email
}
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- Head: processing of some variables.

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- Body: pattern matching expression.

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- Body: pattern matching expression.

We focus on P.

But things can become more complex...

Interesting features of pattern matching on graphs

- Grouping
- Optional parts
- Nesting
- Union of patterns
- Filtering
- ...

```
{ P1
  P2 }
```

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```
{ { P1
    P2 }

  { P3
    P4 }

}
```

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```
{ { P1
    P2
    OPTIONAL { P5 }  }

  { P3
    P4
    OPTIONAL { P7 }  }

}
```

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UNION
{ P9 }
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A formal semantics for SPARQL is needed.

A formal approach would be beneficial

- Clarifying corner cases
- Helping in the implementation process
- Providing sound foundations

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In our work:

- A formal compositional semantics (for simple RDF)
- Normal forms for patterns
- Complexity bounds
- Optimization procedures

Outline

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A standard algebraic syntax

- Triple patterns: just triples + variables, without bnodes

original SPARQL syntax	algebraic syntax
`?X :name "john"`	$(?X, \text{name}, \text{john})$

- Graph patterns: full parenthesized algebra

original SPARQL syntax	algebraic syntax
`{ P1 P2 }`	$(P_1 \text{ AND } P_2)$
`{ P1 OPTIONAL { P2 }}`	$(P_1 \text{ OPT } P_2)$
`{ P1 } UNION { P2 }`	$(P_1 \text{ UNION } P_2)$
`{ P1 FILTER ( R ) }`	$(P_1 \text{ FILTER } R)$

A standard algebraic syntax (cont.)

- Explicit precedence/association

Example

```
{ t1
  t2
  OPTIONAL { t3 }
  OPTIONAL { t4 }
  t5
}
```

$$((((\, t_1 \text{ AND } t_2 \,) \text{ OPT } t_3 \,) \text{ OPT } t_4 \,) \text{ AND } t_5 \,)$$

Mappings: building block for the semantics

Definition

A mapping is a partial function from variables to RDF terms.

The evaluation of a pattern results in a set of mappings.

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The semantics of triple patterns

Given an RDF graph and a triple pattern t

Definition

The evaluation of t is the set of mappings that

- make t to match the graph
- have as domain the variables in t.

Example

graph	triple	evaluation
$(R_1, \text{name}, \text{john})$		
$(R_1, \text{email}, \text{J@ed.ex})$	$(?X, \text{name}, ?Y)$	
$(R_2, \text{name}, \text{paul})$		

	$?X$	$?Y$
μ_1:	R_1	john
μ_2:	R_2	paul

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Compatible mappings: mappings that can be merged.

Definition

Mappings are compatibles if they agree in their shared variables.
Compatible mappings can be extended.

Example

	$?X$	$?Y$	$?U$	$?V$
μ_1 :	R_1	john		
μ_2 :	R_1		J@edu.ex	
μ_3 :			P@edu.ex	R_2

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- μ_2 and μ_3 are not compatible

Sets of mappings and operations

Let M_1 and M_2 be sets of mappings:

Definition

Join: $M_1 \bowtie M_2$

- extending mappings in M_1 with compatible mappings in M_2

Difference: $M_1 \setminus M_2$

- mappings in M_1 that cannot be extended with mappings in M_2

Union: $M_1 \cup M_2$

- mappings in M_1 plus mappings in M_2 (the usual set union)

Definition

Left Outer Join: $M_1 \mathbin{⟕} M_2 = (M_1 \bowtie M_2) \cup (M_1 \setminus M_2)$

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Semantics in terms of operations between evaluations

Let M_1 and M_2 be the evaluation of P_1 and P_2.

Definition

The evaluation of:

$$\begin{array}{rcl} (P_1 \text{ AND } P_2) & \rightarrow & M_1 \bowtie M_2 \\ (P_1 \text{ UNION } P_2) & \rightarrow & M_1 \cup M_2 \\ (P_1 \text{ OPT } P_2) & \rightarrow & M_1 \mathbin{⟕} M_2 \end{array}$$

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Simple example

Example

$(R_1, \text{name}, \text{john})$
$(R_1, \text{email}, \text{J@ed.ex})$
$(R_2, \text{name}, \text{paul})$

$((?X, \text{name}, ?Y) \text{ OPT } (?X, \text{email}, ?E))$

- from the Join
- from the Difference
- from the Union

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Boolean filter expressions (value constraints)

In filter expressions we consider

- equality $=$ among variables and RDF terms
- unary predicate bound
- boolean combinations ($\wedge$, $\vee$, $\neg$)

Satisfaction of value constraints

A mapping satisfies

- $?X = c$ if it gives the value c to variable $?X$
- $?X = ?Y$ if it gives the same value to $?X$ and $?Y$
- bound($?X$) if it is defined for $?X$

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The evaluation of (P FILTER R):

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Natural algebraic properties: a simple normal from

- AND and UNION are commutative and associative.
- AND, OPT, and FILTER distribute over UNION.

Theorem (UNION Normal Form)

Every graph pattern is equivalent to one of the form

$$P_1 \text{ UNION } P_2 \text{ UNION } \cdots \text{ UNION } P_n$$

with P_i UNION–free.

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Outline

The evaluation decision problem

INPUT:

A mapping, a graph pattern, and an RDF graph.

OUTPUT:

Is the mapping in the evaluation of the pattern against the graph?

Evaluation of simple patterns is polynomial.

Theorem

For patterns using only AND *and* FILTER *operators,*

the evaluation problem is polynomial:

$O(\textit{size of the pattern} \times \textit{size of the graph})$.

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the evaluation problem is polynomial:

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Proof.

- Check that the mapping makes every triple to match.
- Then check that the mapping satisfies the FILTERs.

□

Evaluation including UNION is NP-complete.

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- Reduction from propositional SAT.
- The pattern codifies a propositional formula.
- Using $\neg$ bound to codify negation.

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$$\forall x_1 \exists x_2 \forall x_3 \cdots \varphi.$$

- Using nested OPTs to codify quantifier alternations.
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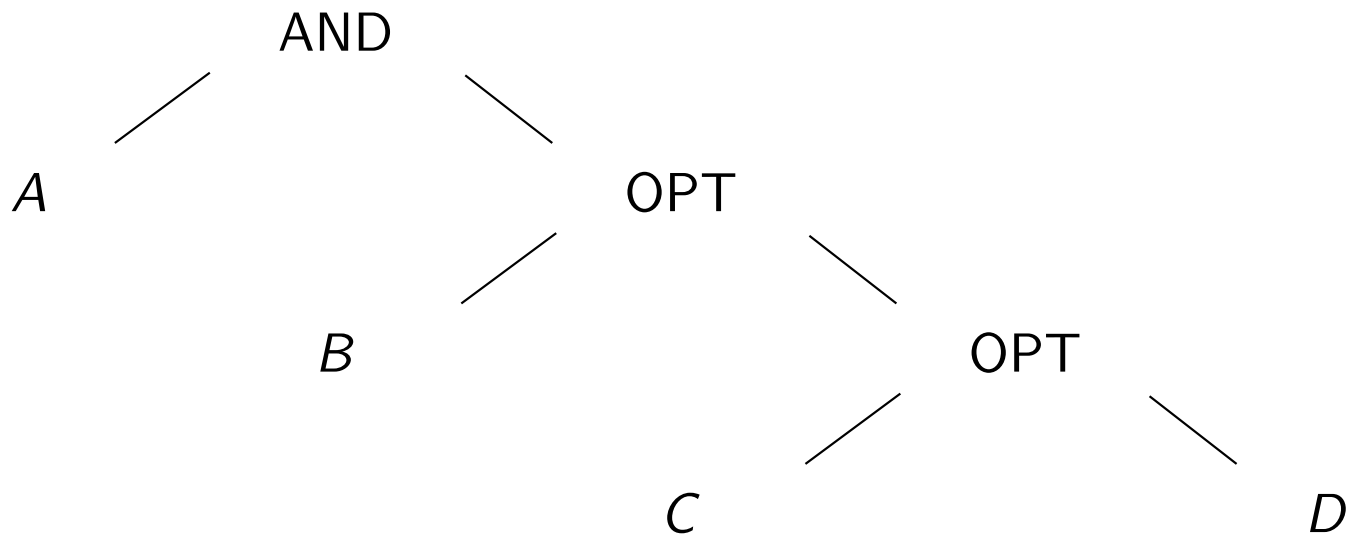
Proof.

From the data–complexity of first–order logic. □

Outline

A procedural semantics: depth–first evaluation

Consider: $(A \text{ AND } (B \text{ OPT } (C \text{ OPT } D)))$



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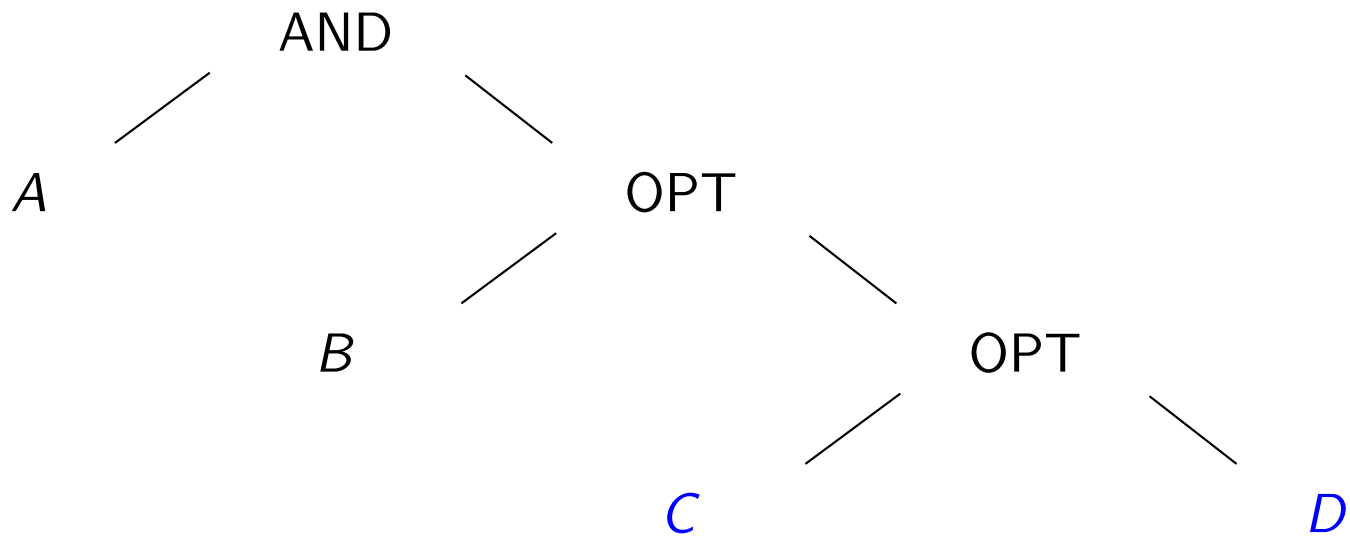

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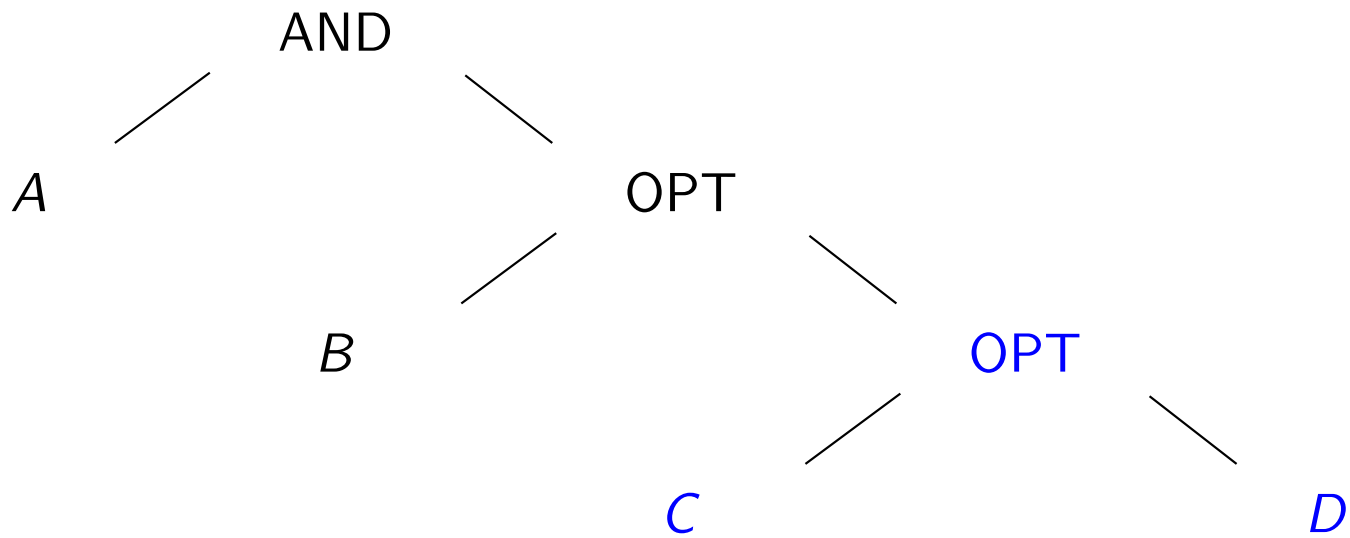

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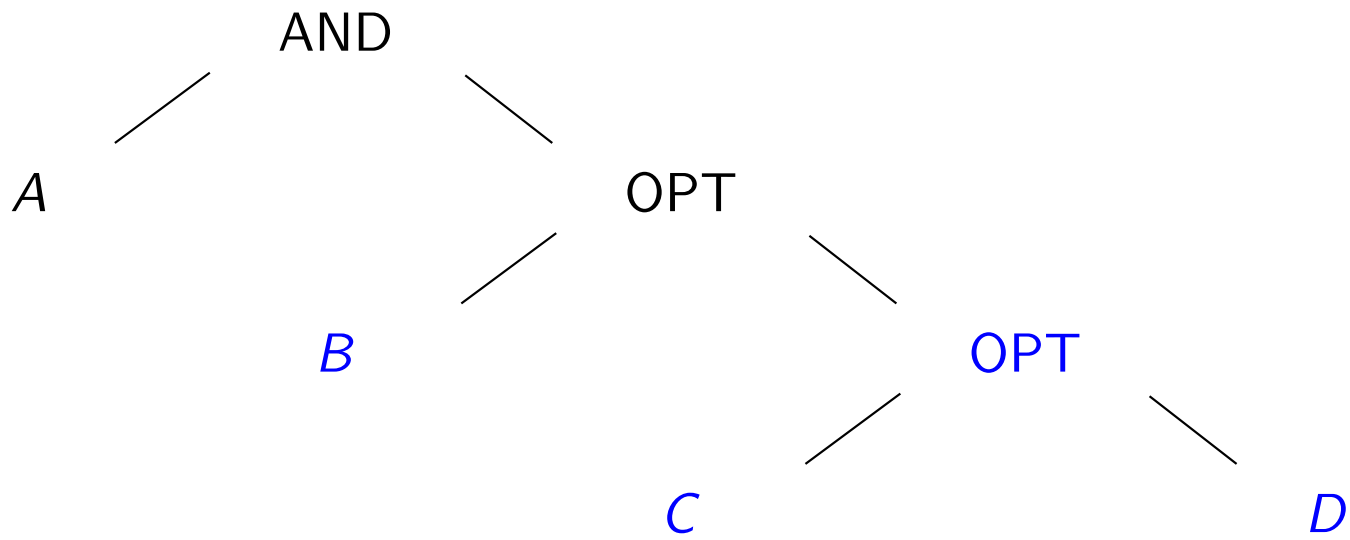

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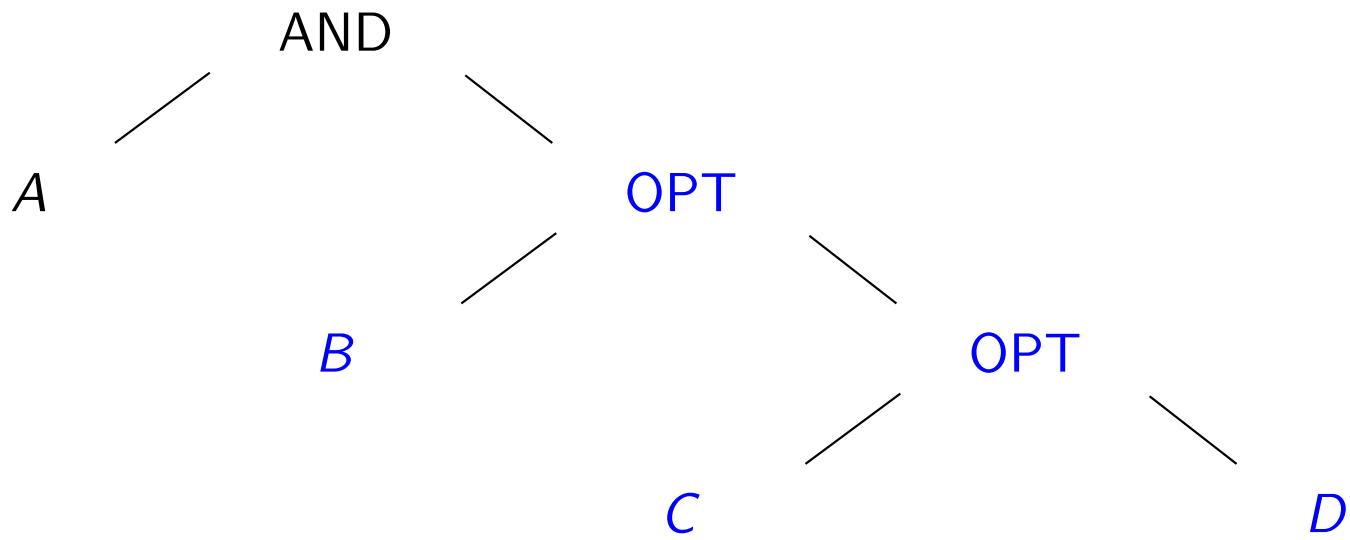

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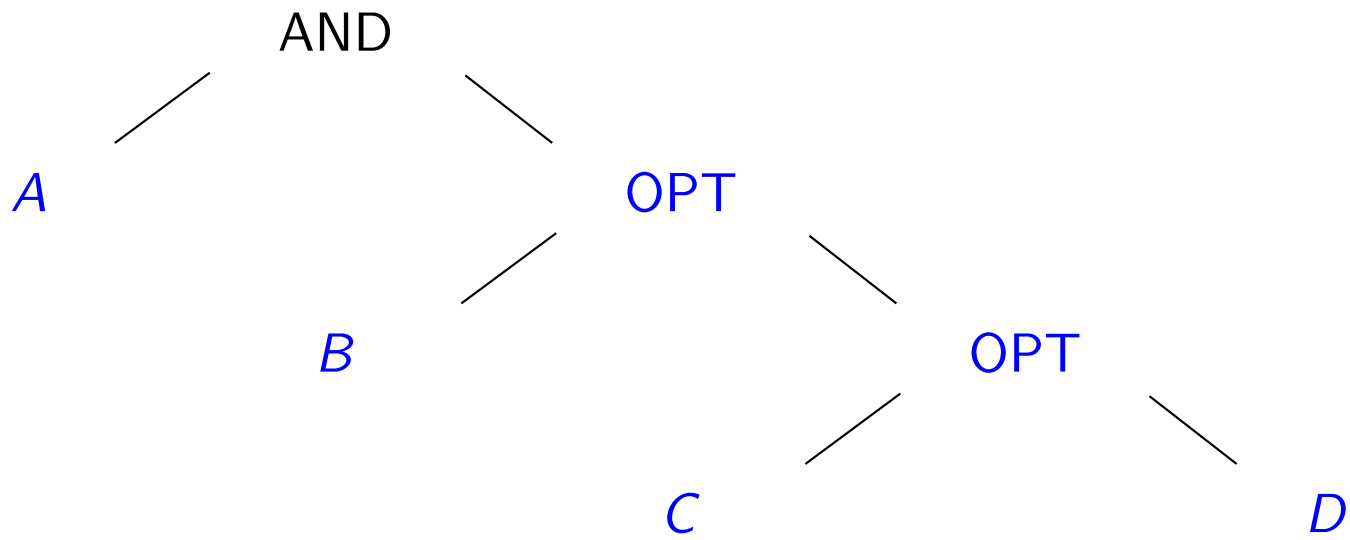

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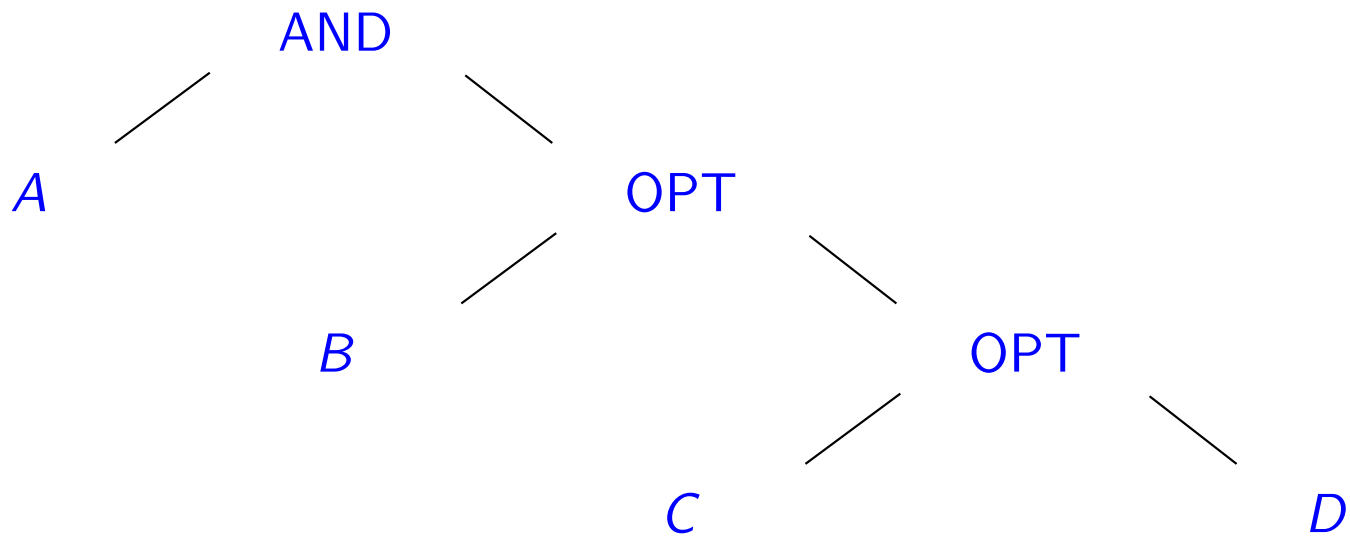

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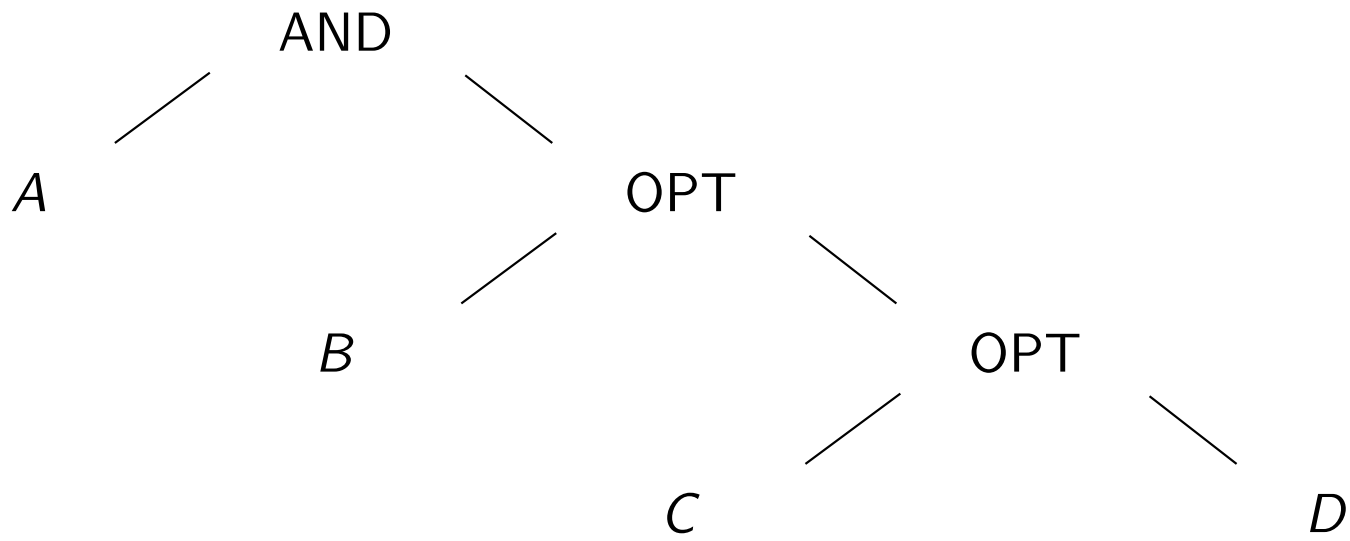

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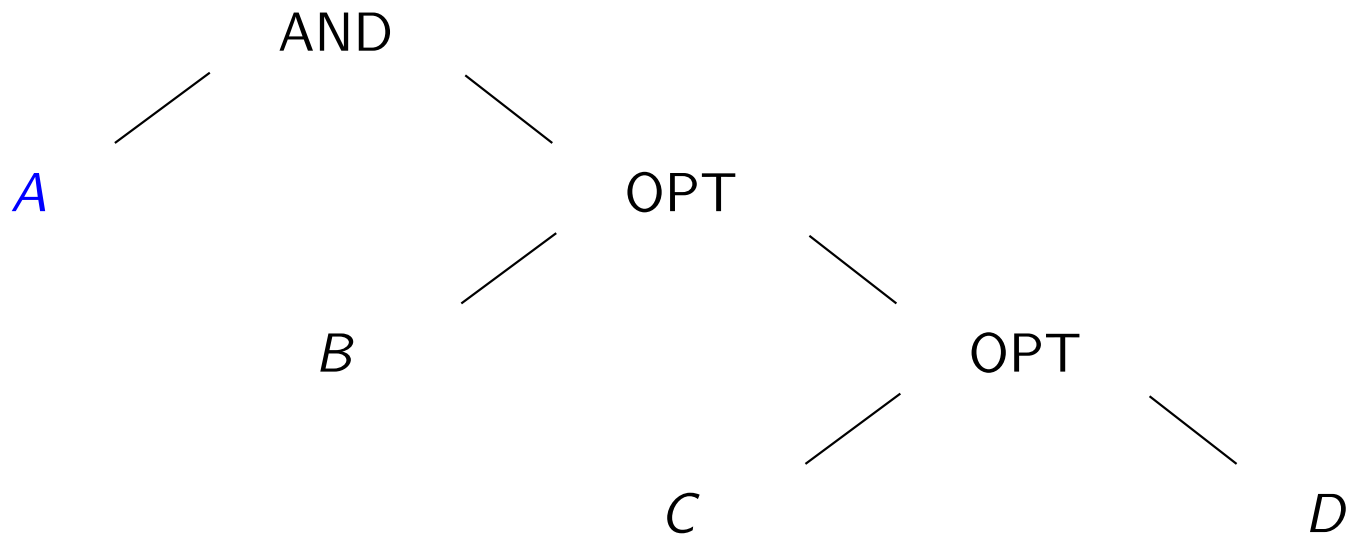

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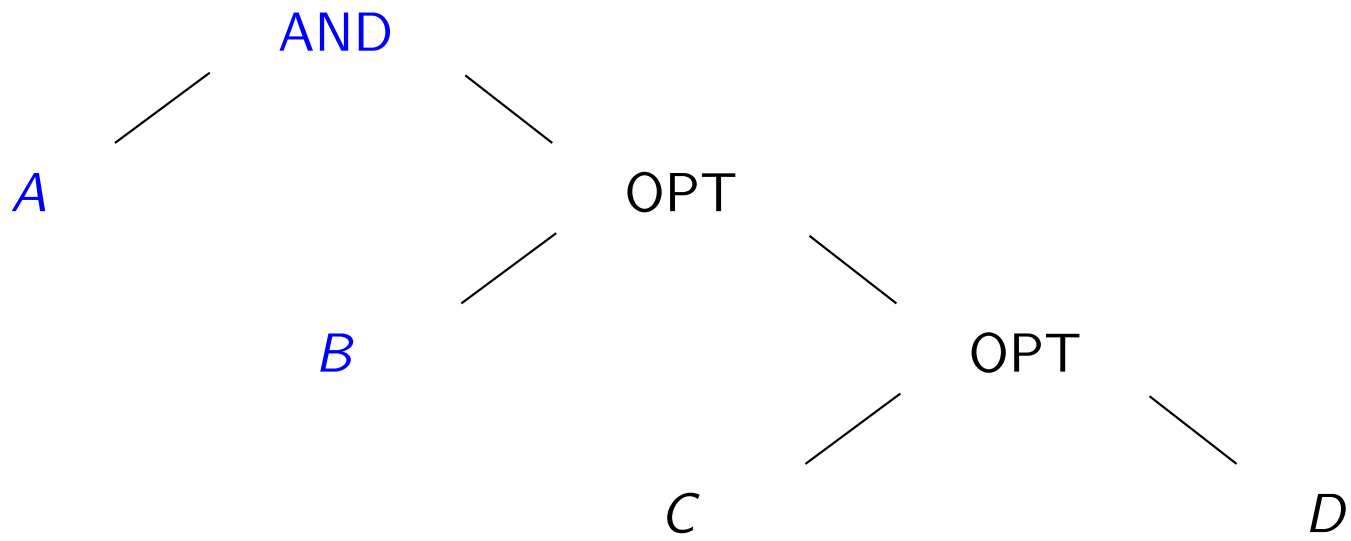

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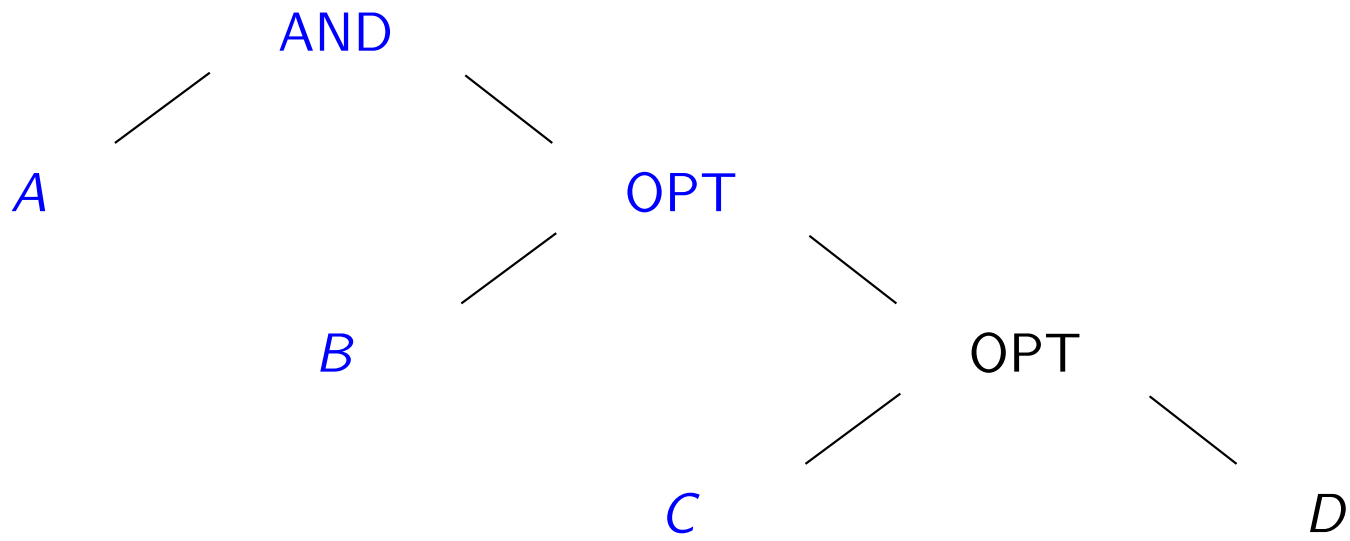

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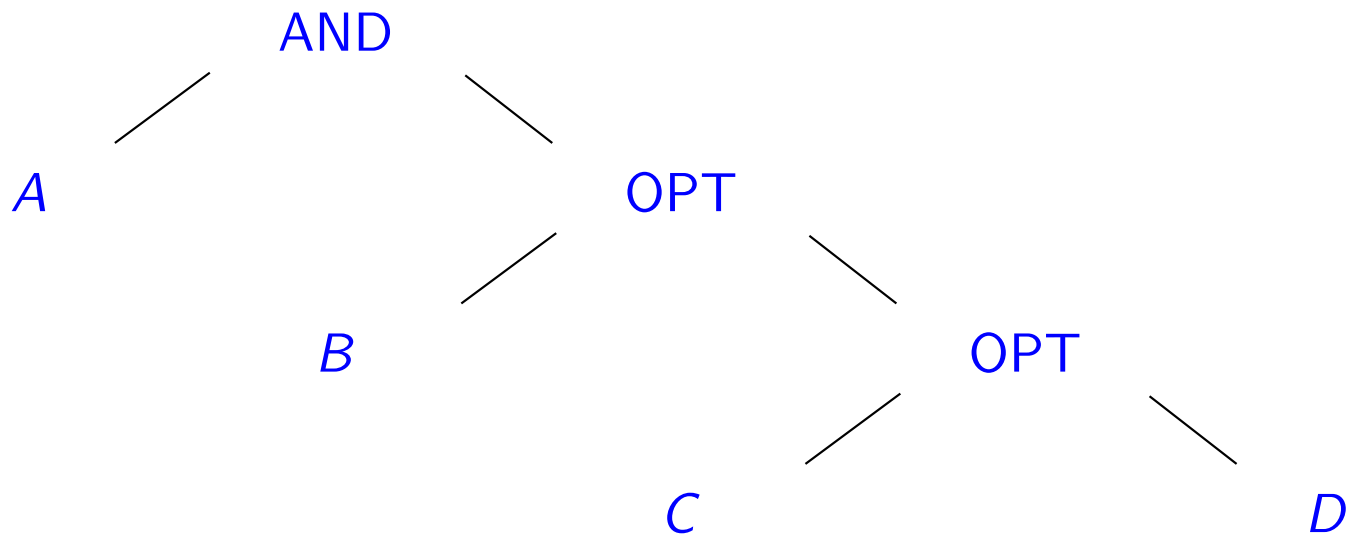

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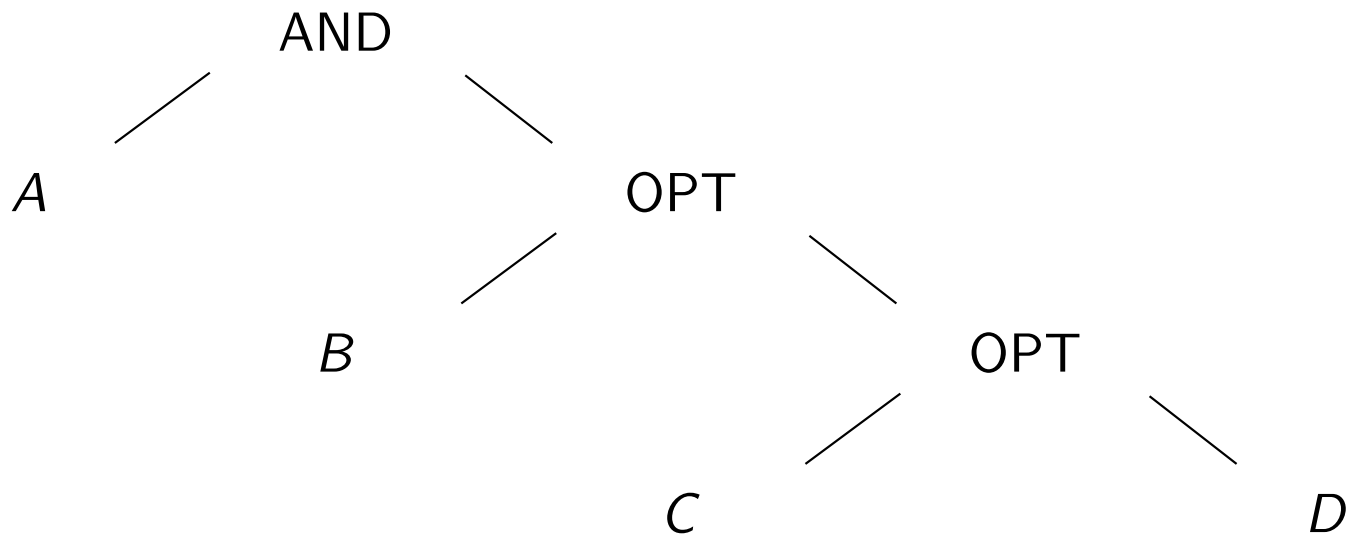

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