

Q_1 : Find the k -th smallest element between indexes i and j

3	3	9	1	2	1	7	6	4	8	9	4	3	7	5	9	2	7	3	5	1	3
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

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3	3	9	1	2	1	7	6	4	8	9	4	3	7	5	9	2	7	3	5	1	3
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

4th smallest element?

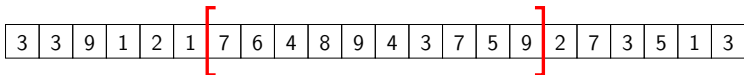
Q_1 : Find the k -th smallest element between indexes i and j

3	3	9	1	2	1	7	6	4	8	9	4	3	7	5	9	2	7	3	5	1	3
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

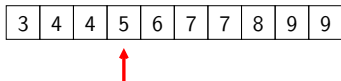
4th smallest element?

3	4	4	5	6	7	7	8	9	9
---	---	---	---	---	---	---	---	---	---

Q_1 : Find the k -th smallest element between indexes i and j



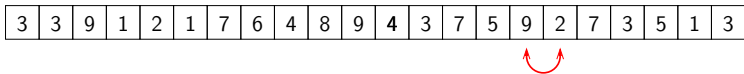
4th smallest element?



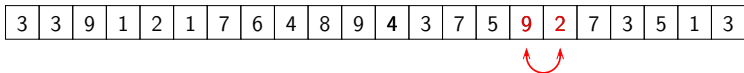
Q₂: Swap the elements at positions i and $i + 1$

3	3	9	1	2	1	7	6	4	8	9	4	3	7	5	9	2	7	3	5	1	3
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

Q₂: Swap the elements at positions i and $i + 1$



Q₂: Swap the elements at positions i and $i + 1$



Q₂: Swap the elements at positions i and $i + 1$

3	3	9	1	2	1	7	6	4	8	9	4	3	7	5	2	9	7	3	5	1	3
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

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3	3	9	1	2	1	7	6	4	8	9	4	3	7	5	2	9	7	3	5	1	3
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

4th smallest element?

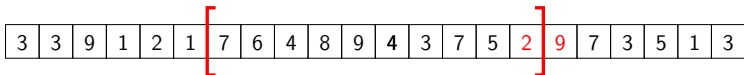
Q₂: Swap the elements at positions i and $i + 1$

3	3	9	1	2	1	7	6	4	8	9	4	3	7	5	2	9	7	3	5	1	3
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

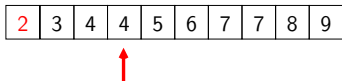
4th smallest element?

2	3	4	4	5	6	7	7	8	9
---	---	---	---	---	---	---	---	---	---

Q₂: Swap the elements at positions i and $i + 1$



4th smallest element?



Input

- ▶ sequence of length $N = 10^6$
- ▶ values in the range $[-10^9, 10^9]$
- ▶ 10^5 (online) queries of either type Q_1 or Q_2

Task

- ▶ report the answer to queries of type Q_1 considering the updates Q_2

Wavelet Trees for Competitive Programming

Robinson Castro, Nico Lehmann, Jorge Pérez, Bernardo Subercaseaux

Universidad de Chile, École Centrale Paris

Wavelet Tree

- ▶ represents sequences over a (potentially big) alphabet Σ
- ▶ basic (tree) structure can be built in $O(N \times \log |\Sigma|)$
- ▶ answer several interesting queries in time $O(\log |\Sigma|)$

Wavelet Tree

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- ▶ basic (tree) structure can be built in $O(N \times \log |\Sigma|)$
- ▶ answer several interesting queries in time $O(\log |\Sigma|)$
- ▶ can be smoothly composed with more classical structures

3 SPOJ problems solvable with Wavelet Trees

	Submitted	Accepted
ILKQ1	49	9
ILKQ2	32	6
ILKQ3	35	2

Submissions until April 2016

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The Wavelet Tree

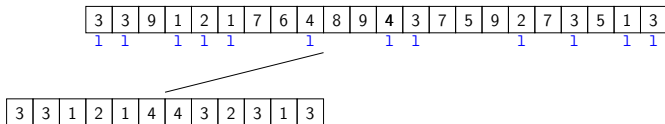
Wavelet Tree construction

3	3	9	1	2	1	7	6	4	8	9	4	3	7	5	9	2	7	3	5	1	3
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

Wavelet Tree construction

3	3	9	1	2	1	7	6	4	8	9	4	3	7	5	9	2	7	3	5	1	3
1	1		1	1	1			1			1	1			1		1			1	1

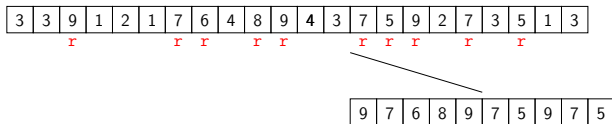
Wavelet Tree construction



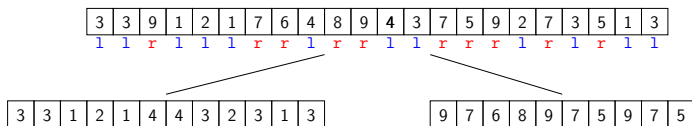
Wavelet Tree construction

3	3	9	1	2	1	7	6	4	8	9	4	3	7	5	9	2	7	3	5	1	3
		r				r	r		r	r		r	r	r		r		r		r	

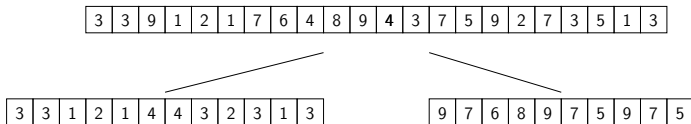
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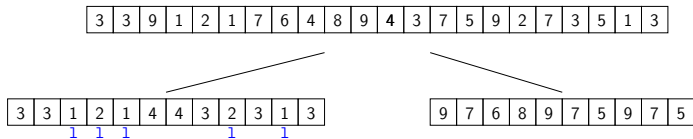
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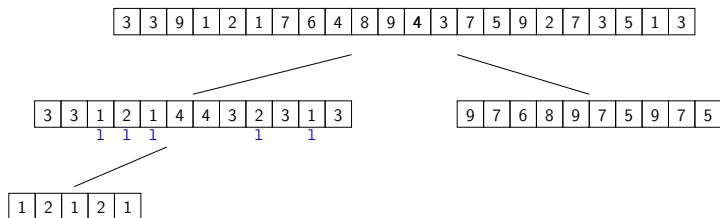
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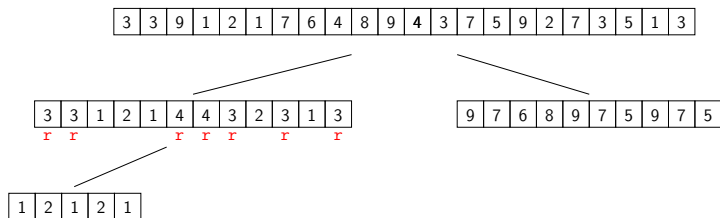
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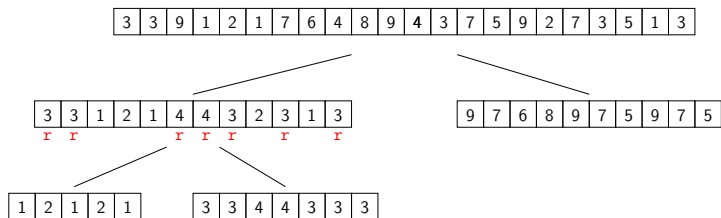
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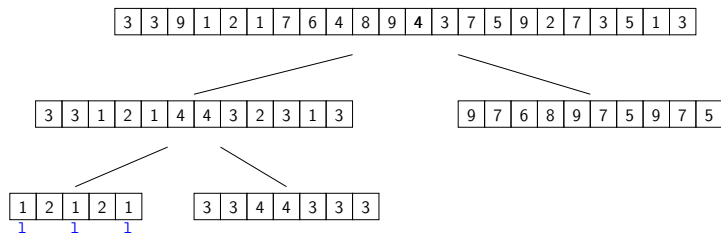
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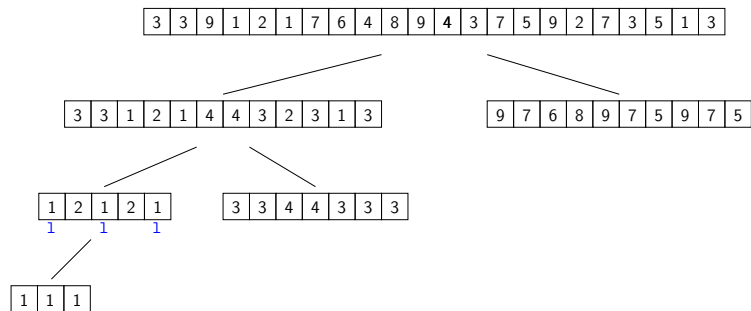
Wavelet Tree construction



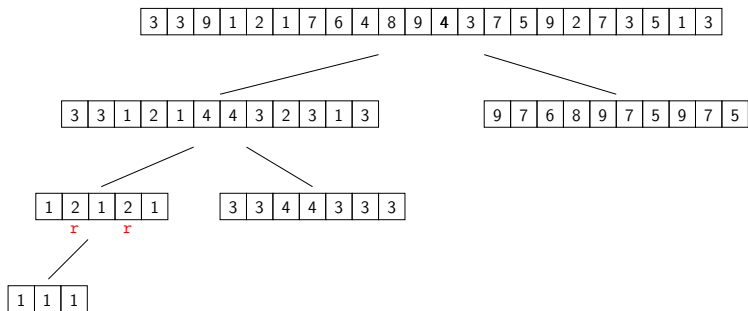
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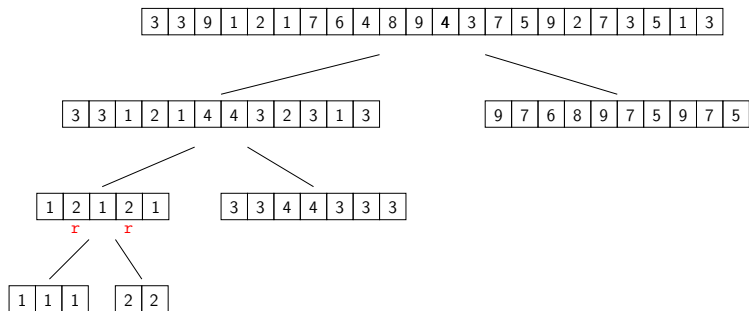
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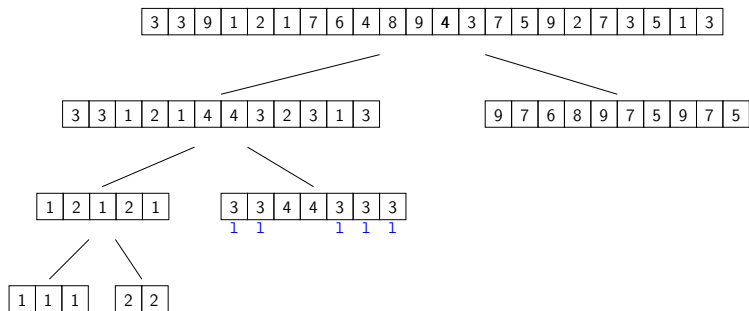
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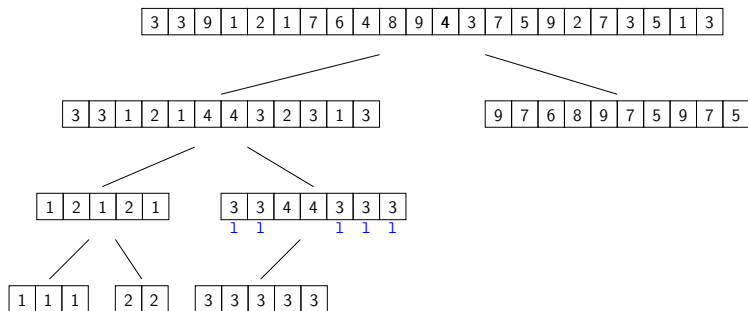
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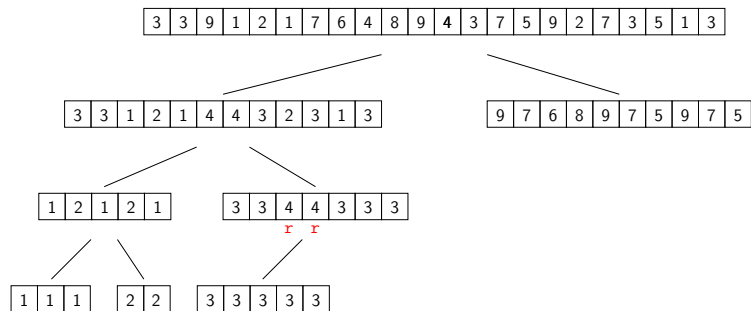
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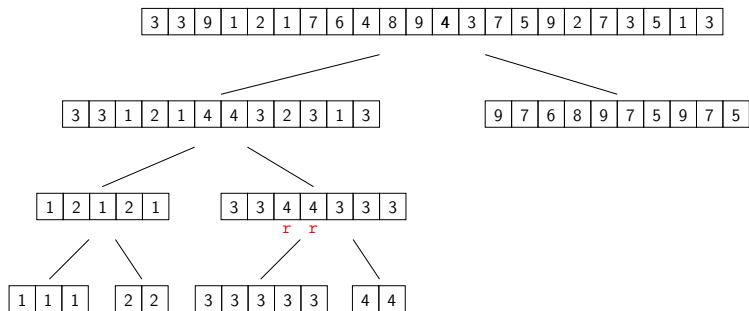
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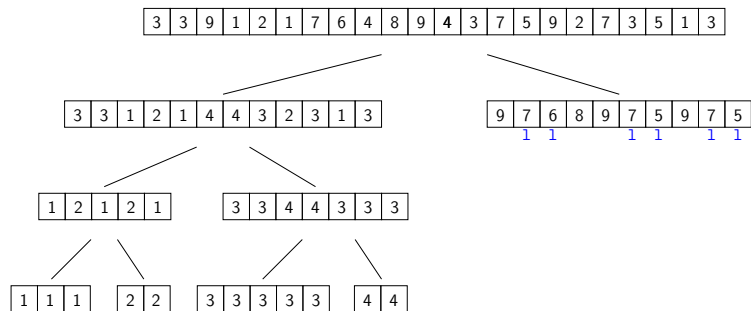
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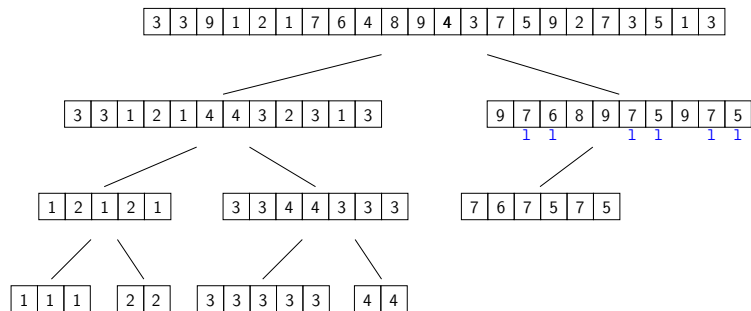
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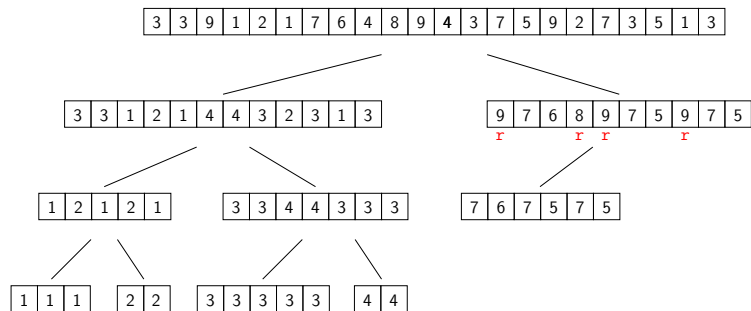
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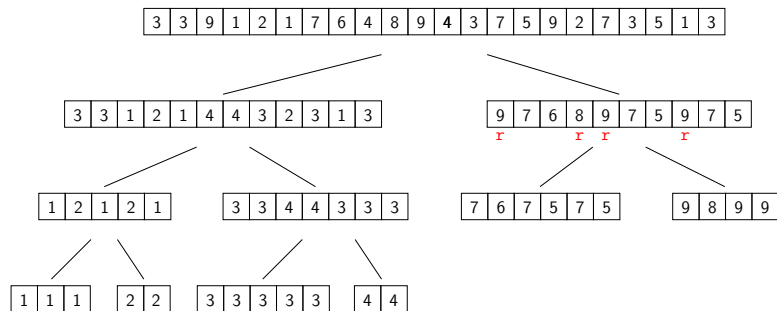
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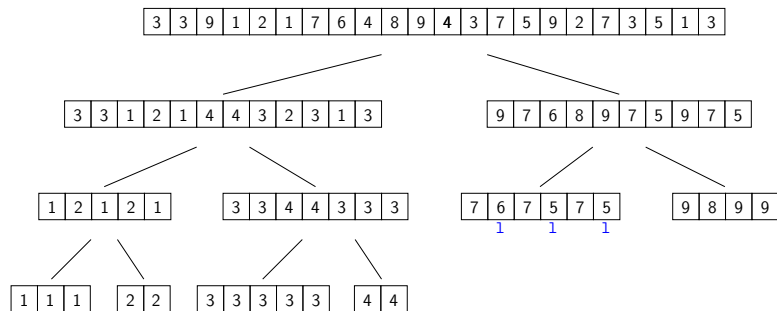
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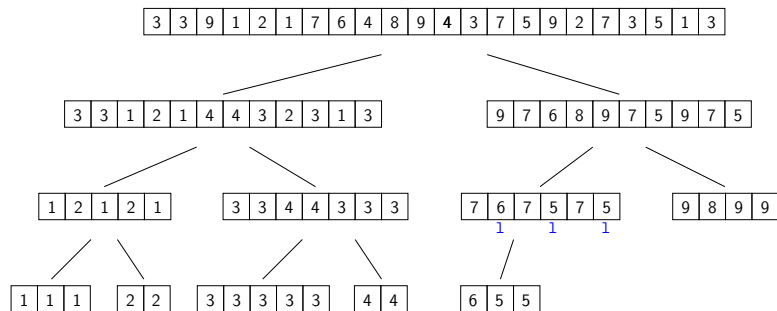
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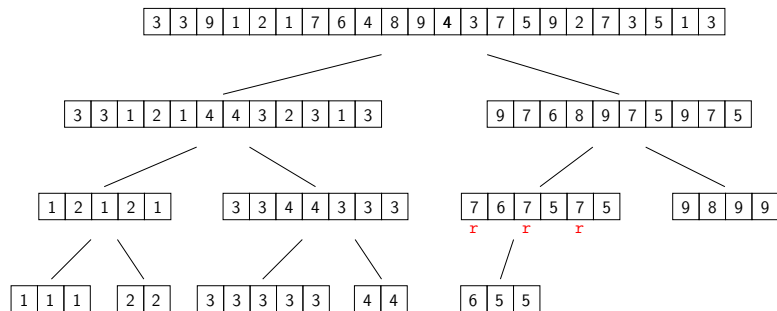
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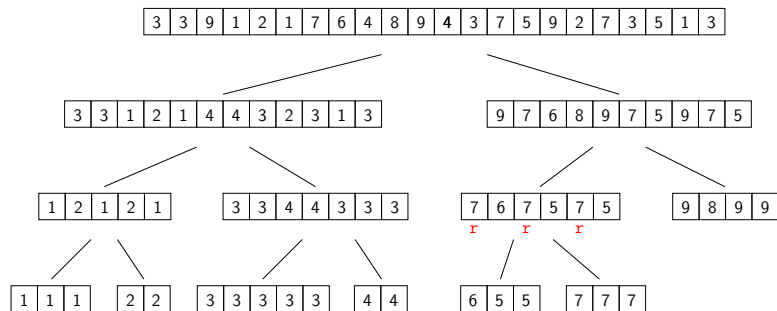
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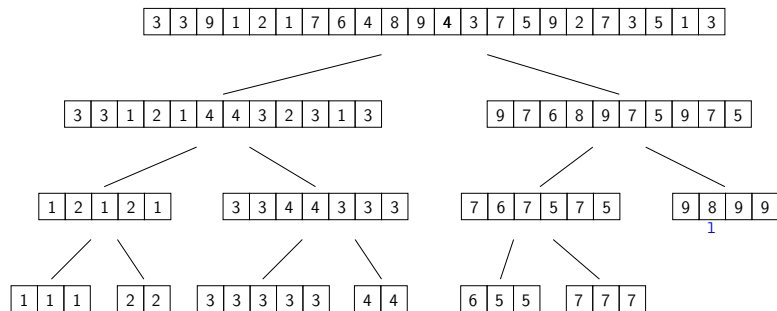
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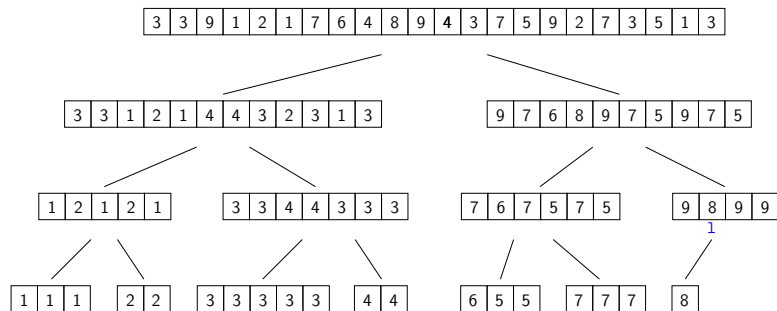
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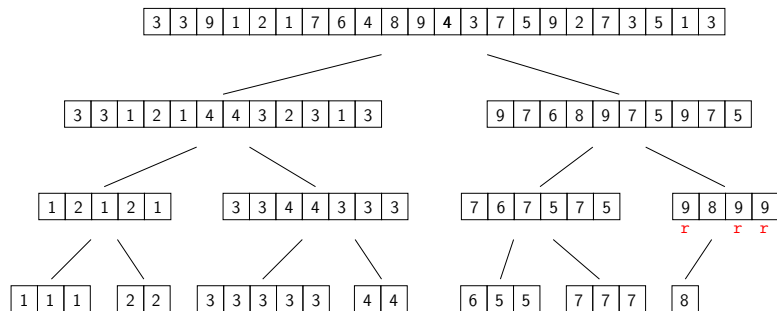
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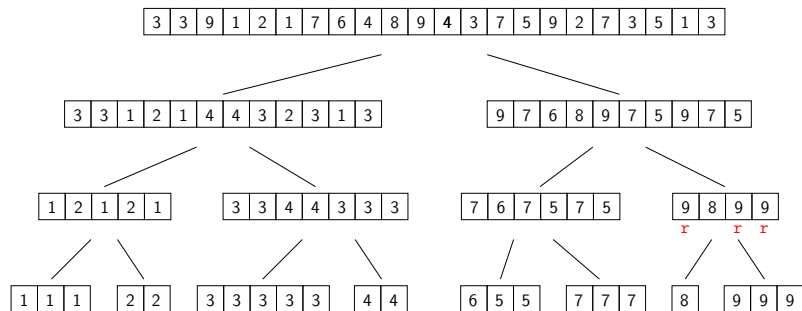
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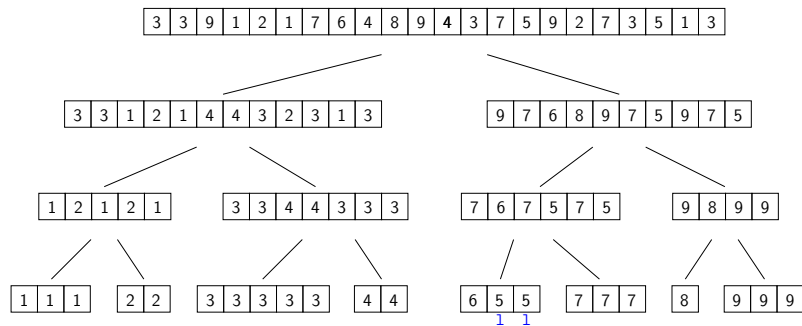
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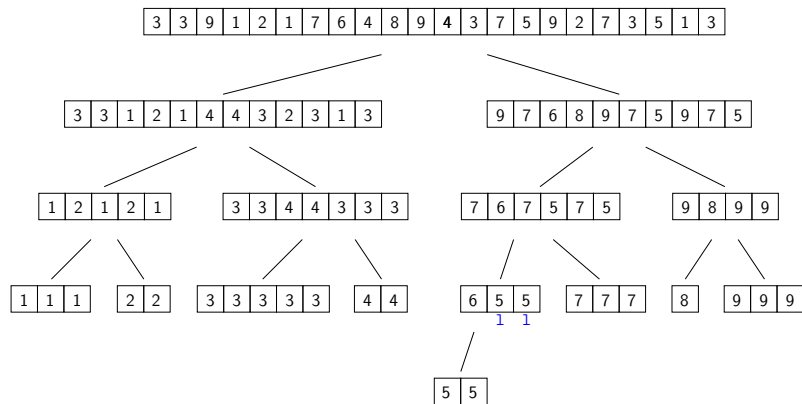
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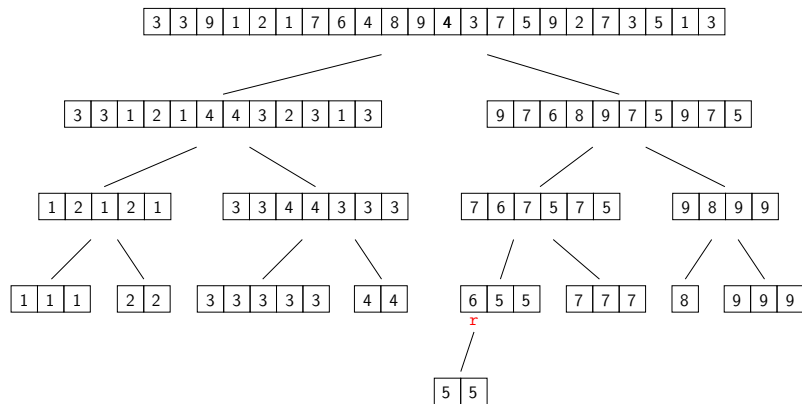
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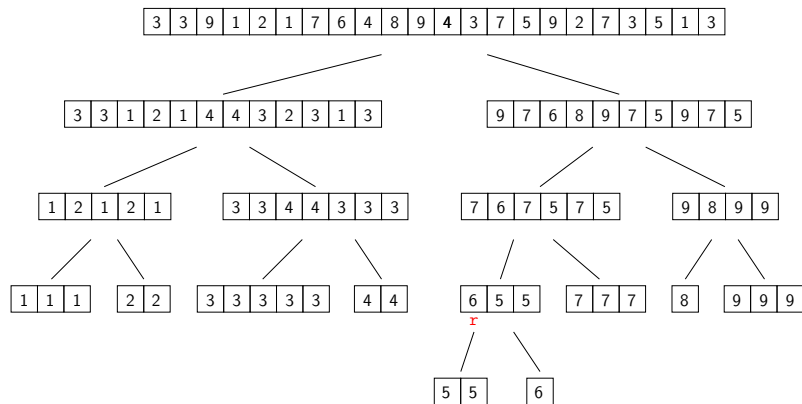
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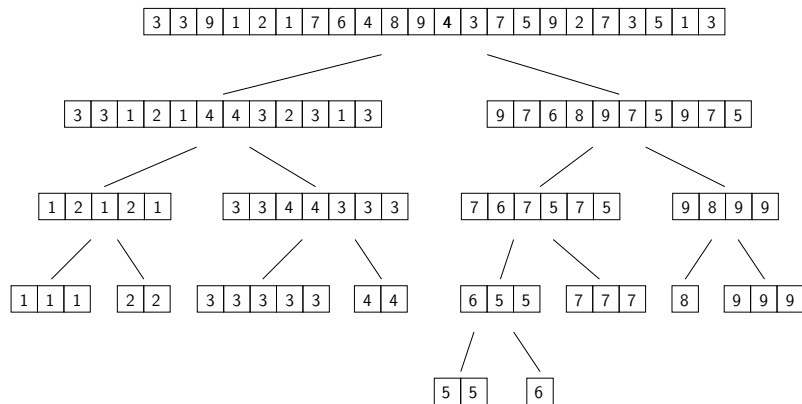
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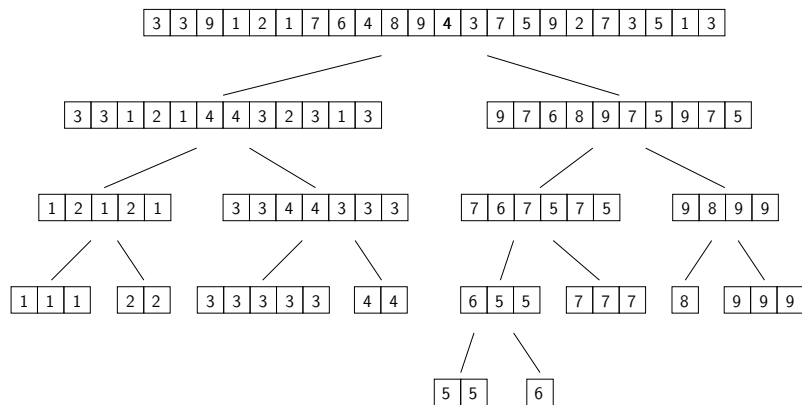
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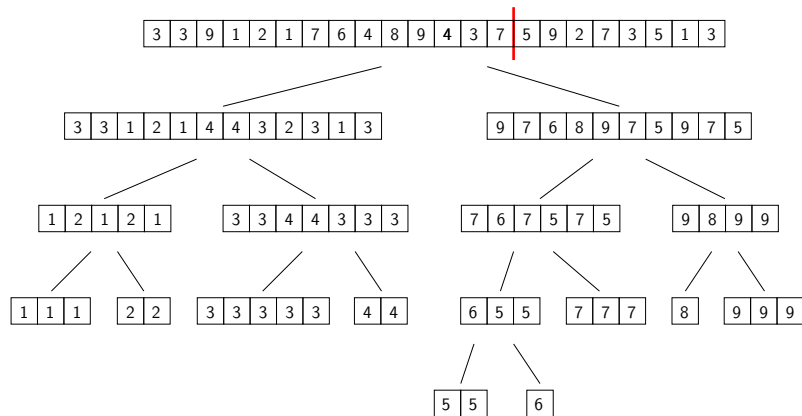
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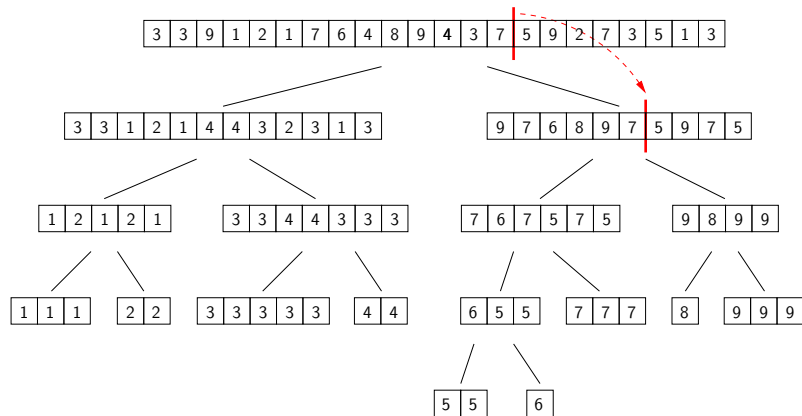
Mapping indexes to the left and to the right



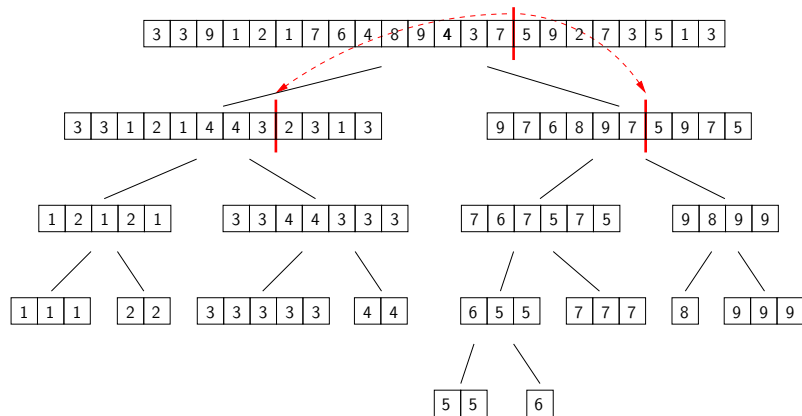
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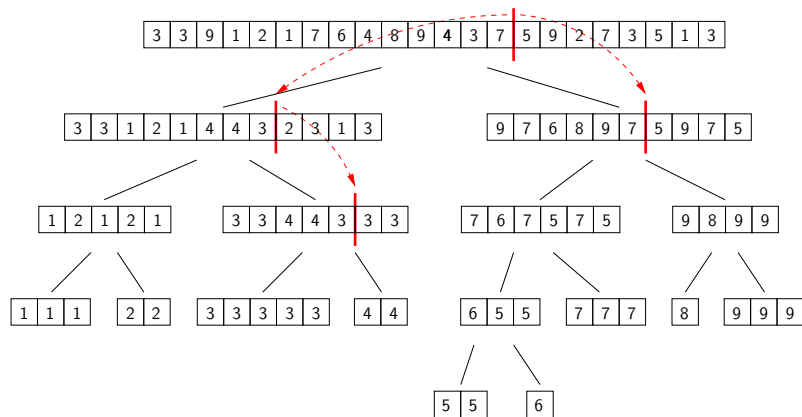
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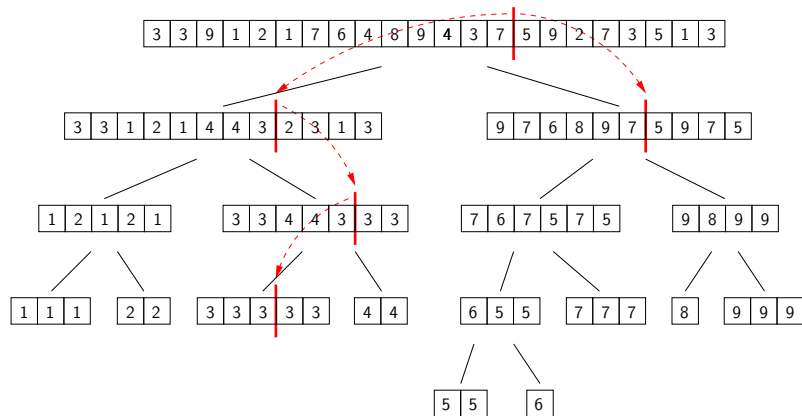
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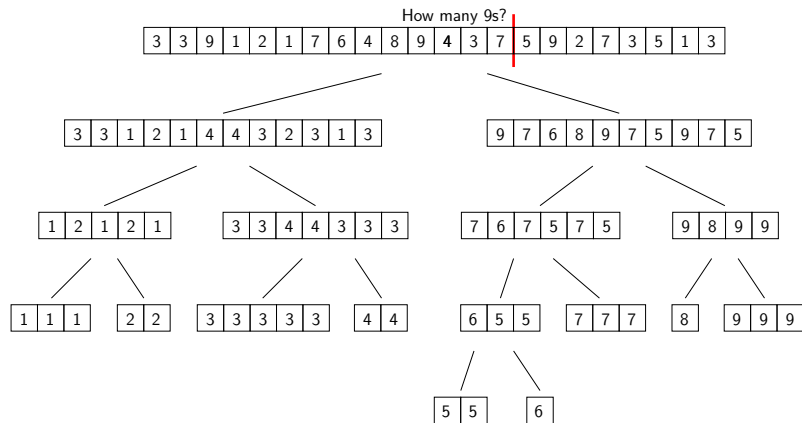


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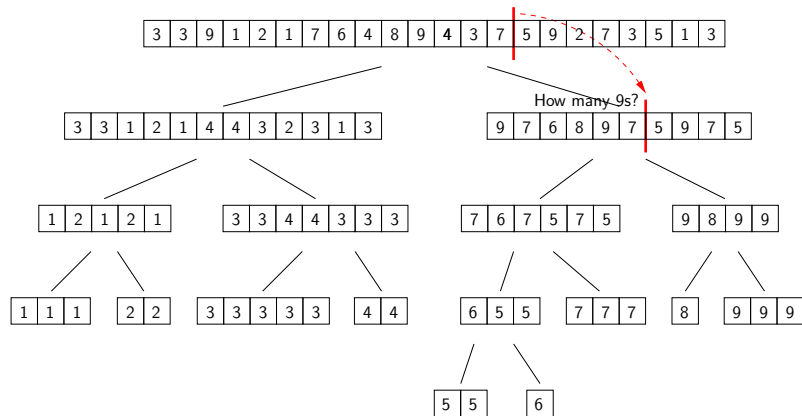


Queries

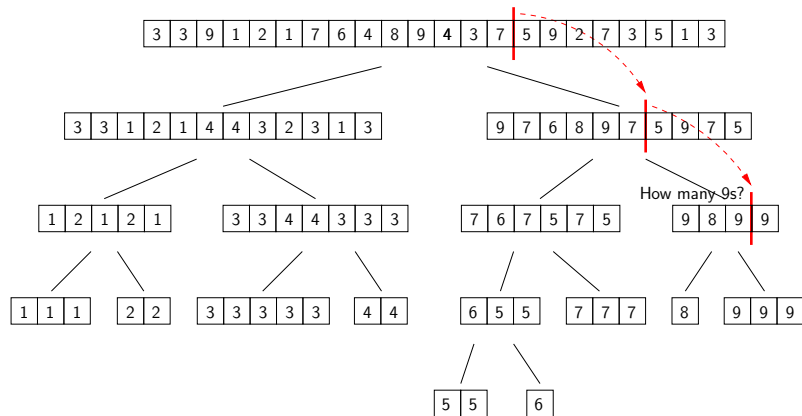
Rank queries



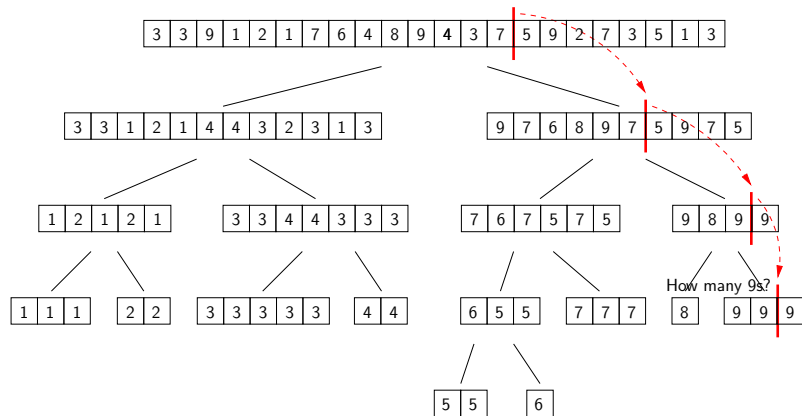
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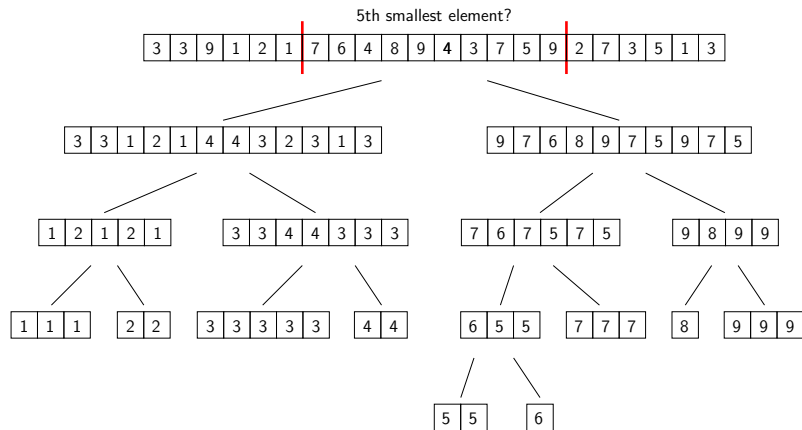
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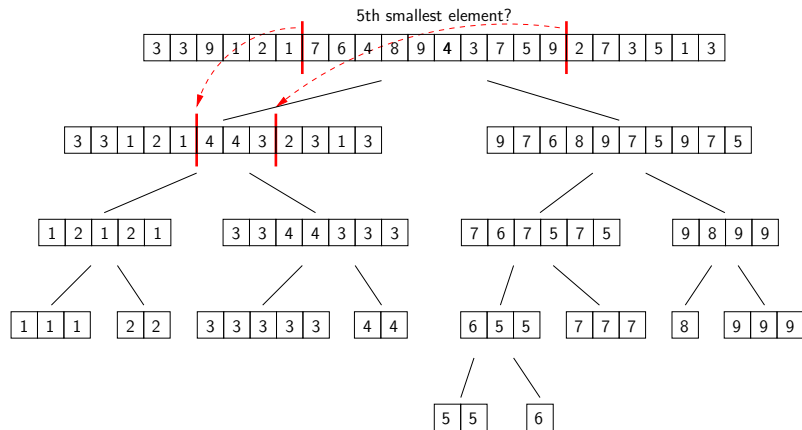
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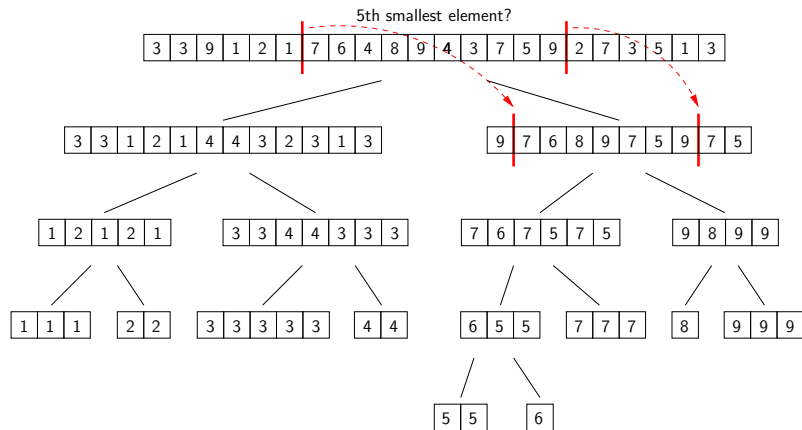
Quantile queries



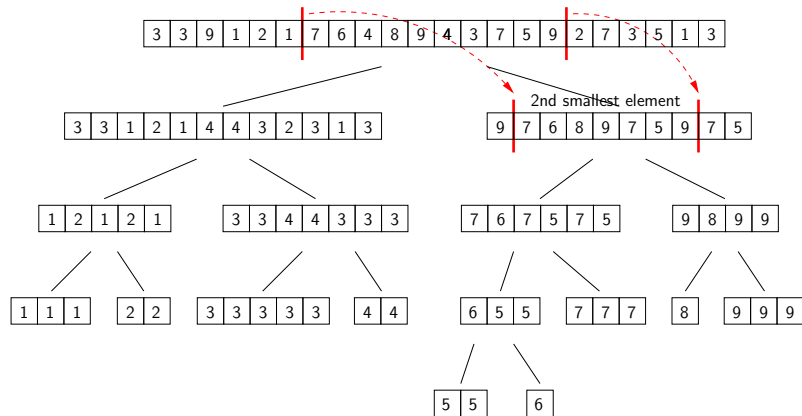
Quantile queries



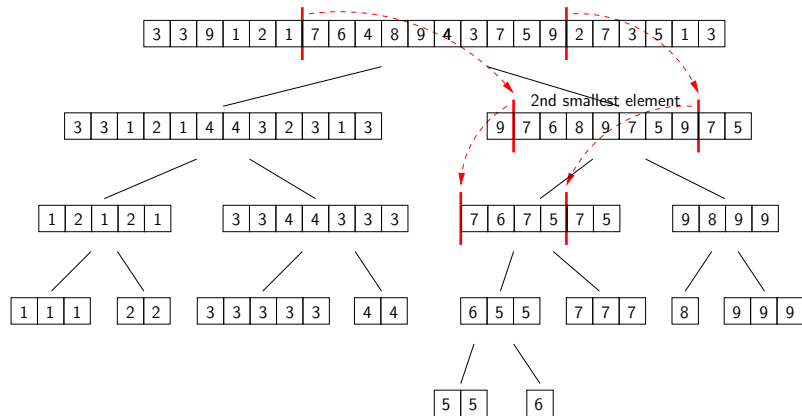
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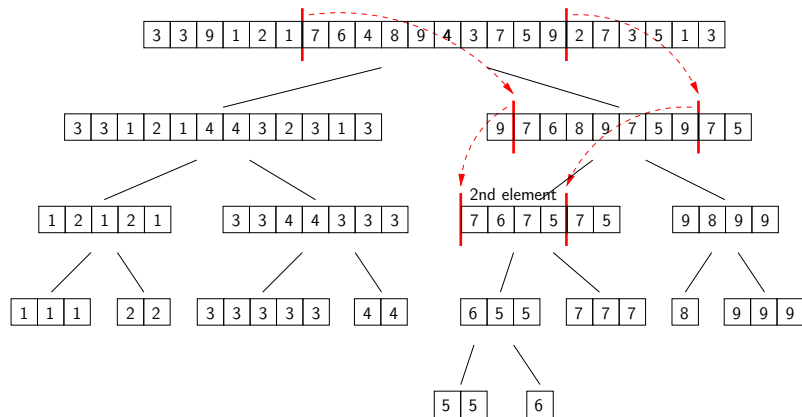
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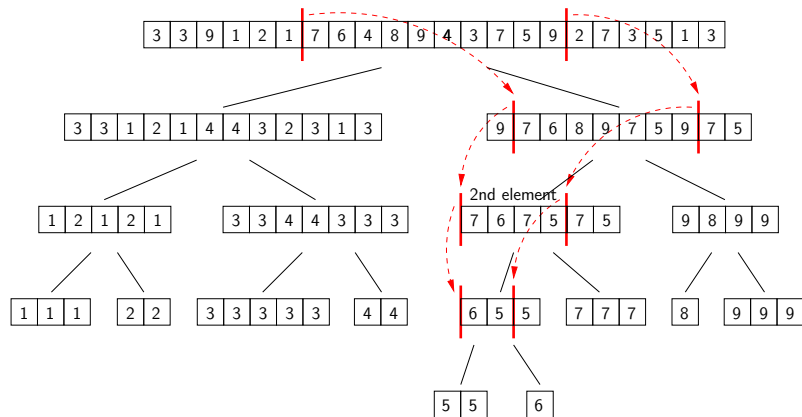
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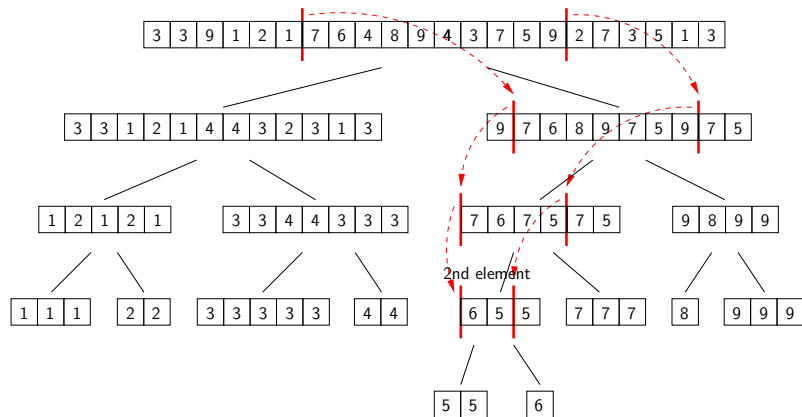
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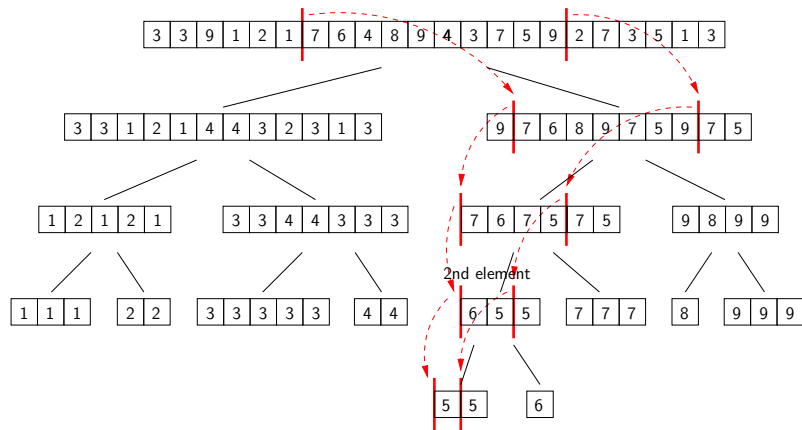
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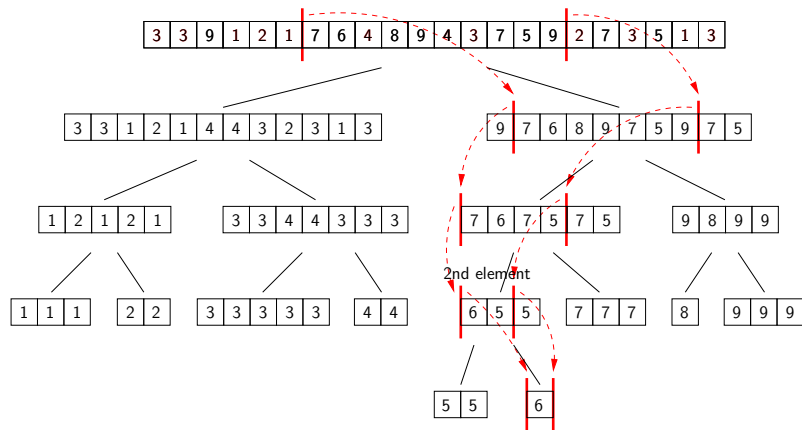
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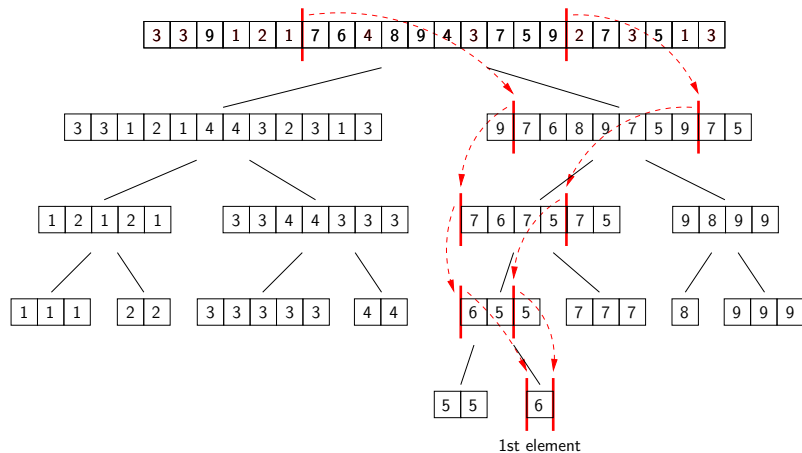
Quantile queries



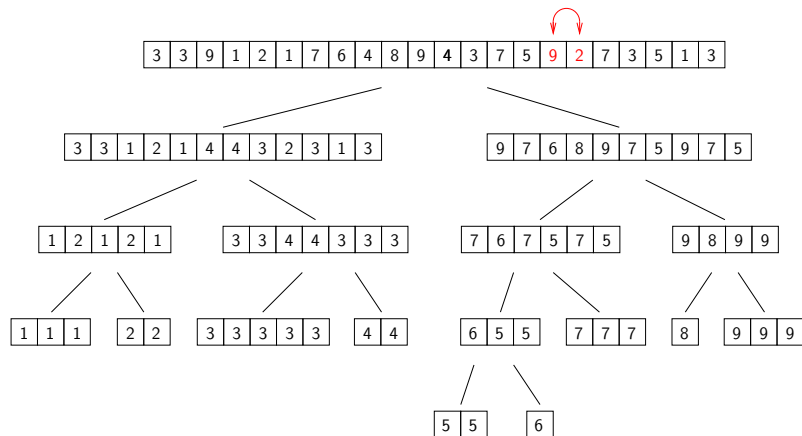
Quantile queries



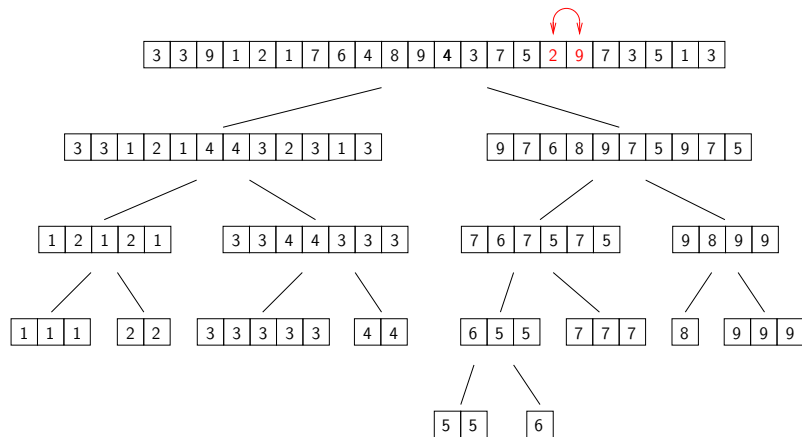
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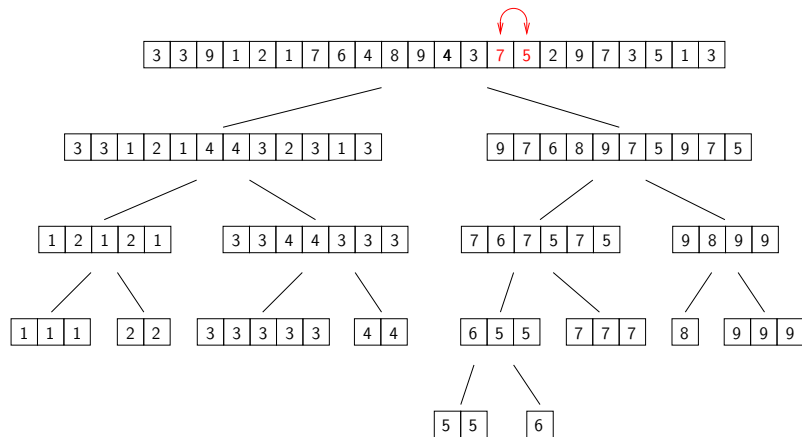
Contiguous Swap



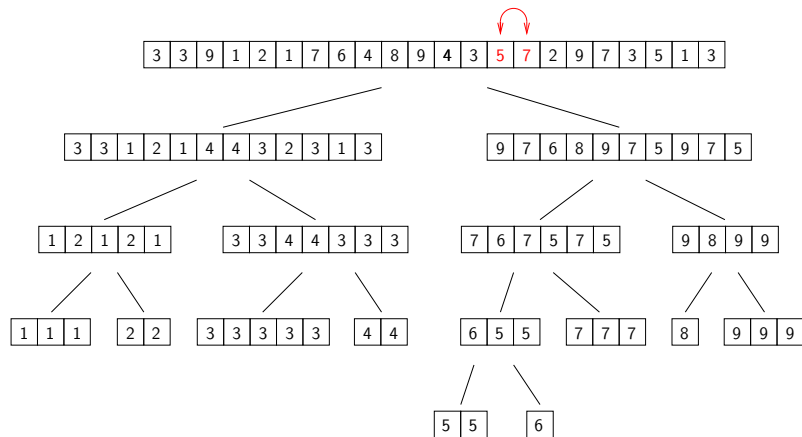
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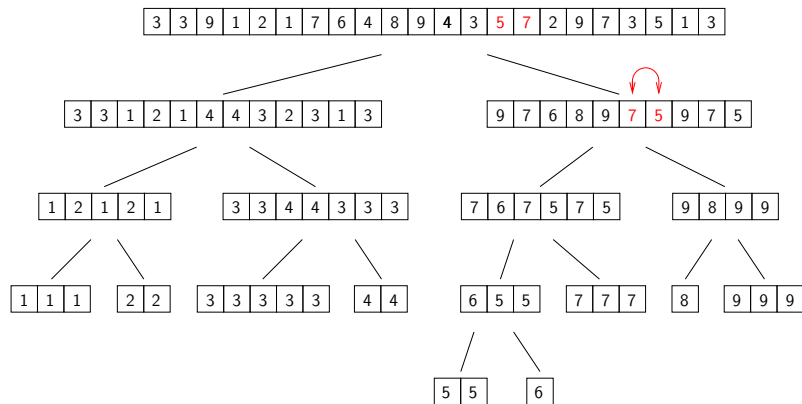
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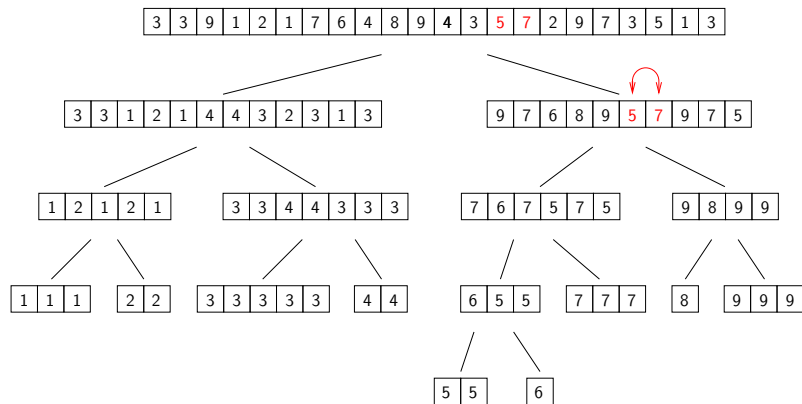
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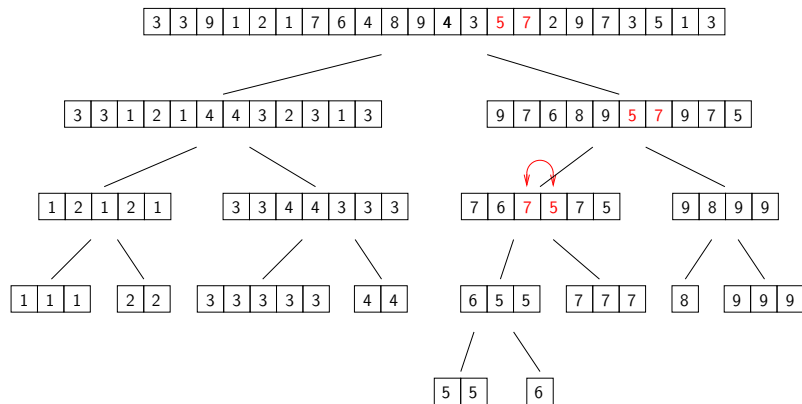
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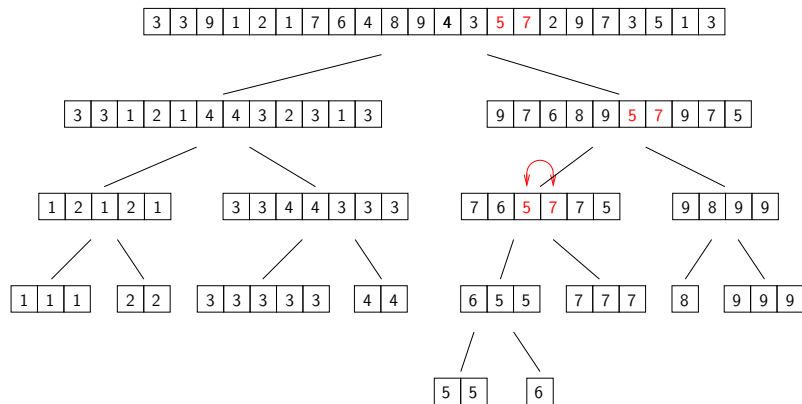
Contiguous Swap



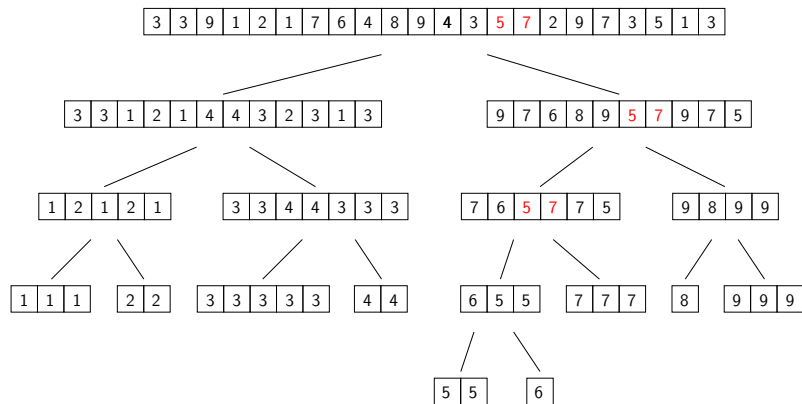
Contiguous Swap



Contiguous Swap



Contiguous Swap



Wavelet Trees support many other queries

- ▶ How many elements with values between a and b ?
- ▶ Rectangular queries in a grid
- ▶ Augment with other structures in every level

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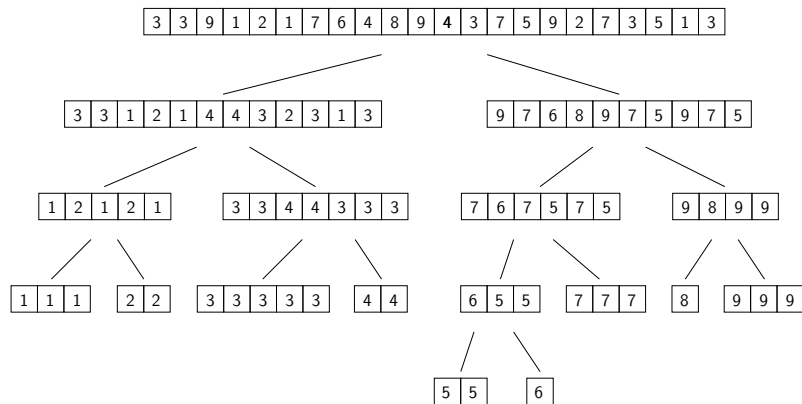
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- ▶ Augment with other structures in every level
- ▶ Push/pop back/front

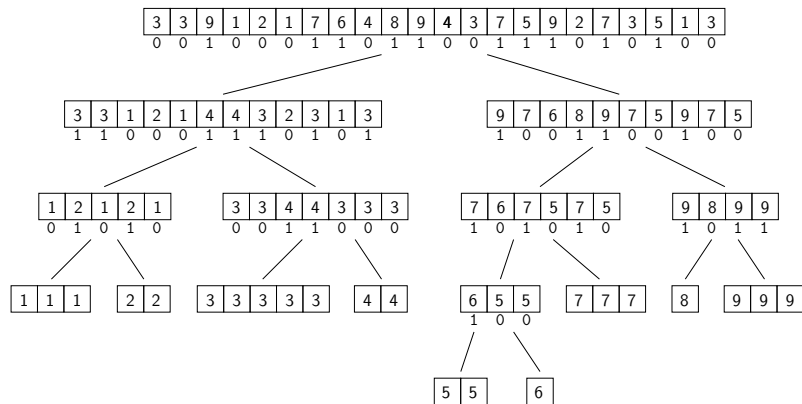
and with a bit more effort

- ▶ Update the value of arbitrary elements in the list

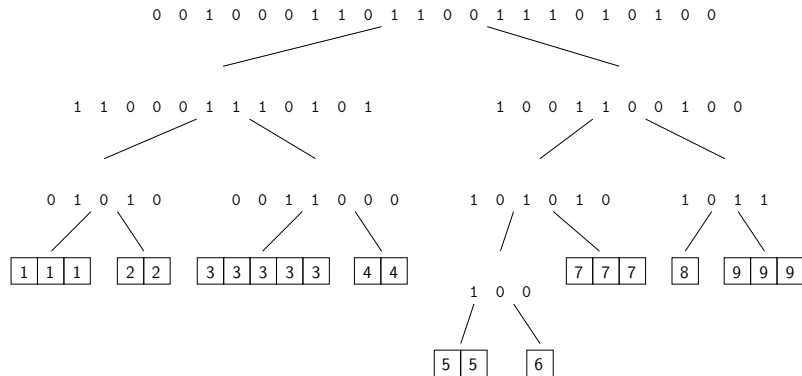
Wavelet Tree implementation



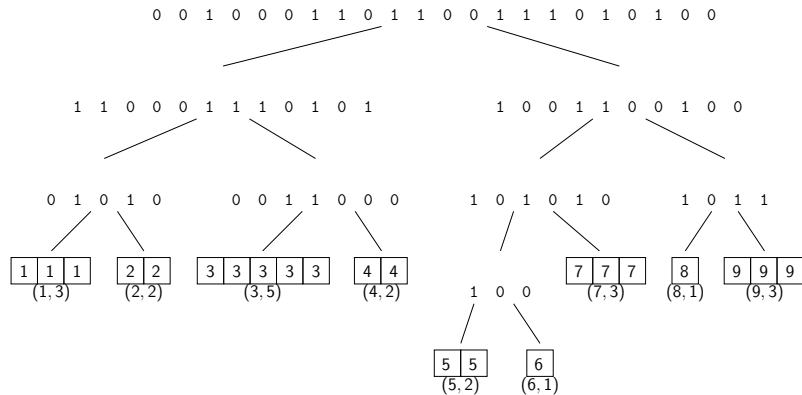
Wavelet Tree implementation



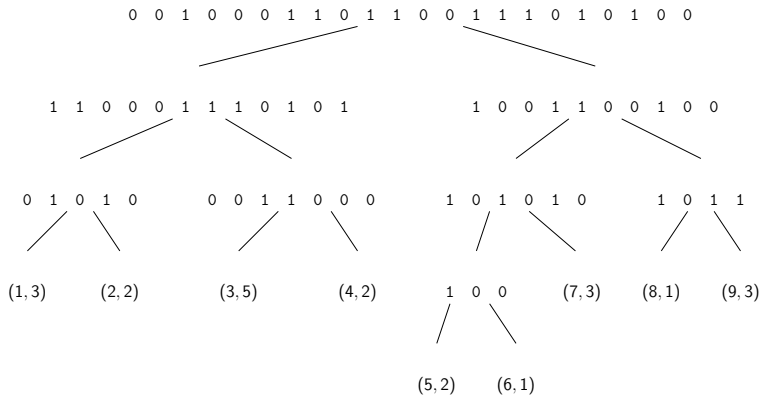
Wavelet Tree implementation



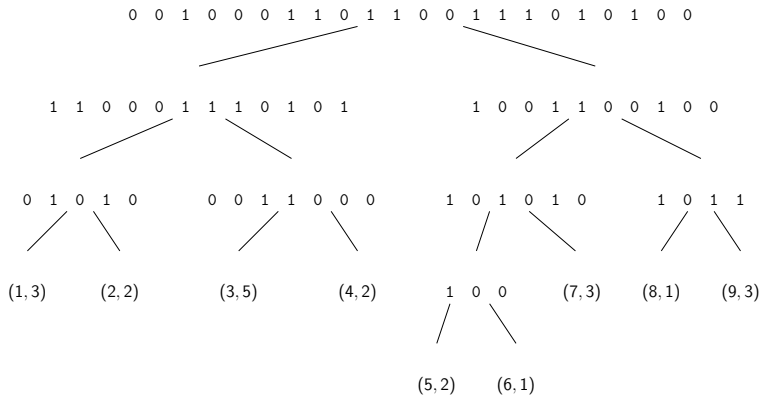
Wavelet Tree implementation



Wavelet Tree implementation



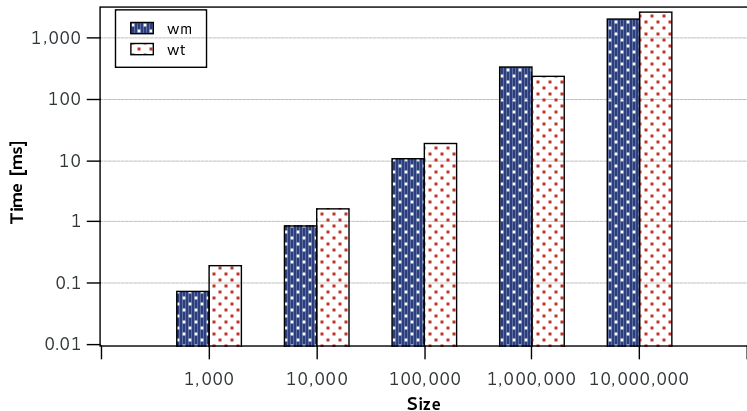
Wavelet Tree implementation



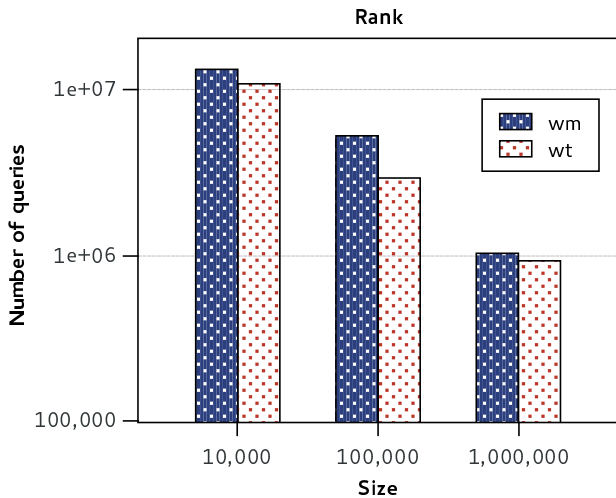
<https://github.com/nilehmann/wavelet-tree>

Performance

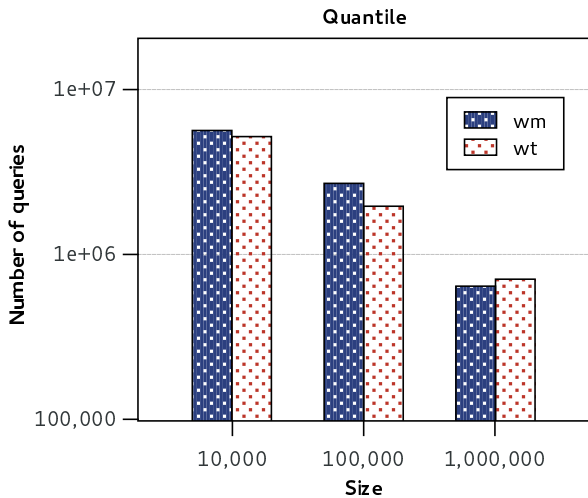
Construction



200ms for input of size 10^6
2s for input of size 10^7

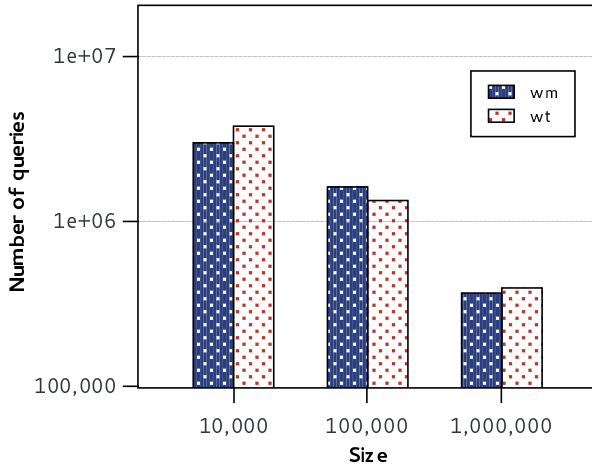


10^6 rank queries in 1s for input of size 10^6 .



$\sim 10^6$ quantile queries in 1s for input of size 10^6 .

Range Counting



$\sim 10^5$ range queries in 1s for input of size 10^6 .

	Submitted	Accepted
ILKQ1	49	9
ILKQ2	32	6
ILKQ3	35	2

Submissions until April 2016

	Submitted	Accepted
ILKQ1	49	9
ILKQ2	32	6
ILKQ3	35	2

Submissions until April 2016

As of today:

	Submitted	Accepted
ILKQ1	49	9
ILKQ2	32	6
ILKQ3	35	2

Submissions until April 2016

As of today: two users submitted correct solutions using Wavelet Trees!

Wavelet Trees for Competitive Programming

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Outline

Motivation

The Wavelet Tree

Queries

Implementation

Performance

Conclusions