Union and Intersection of Schema Mappings

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CliA:

name	balance	city

CliB:

name	amount	account

CliAB:

id	client	balance	account	office

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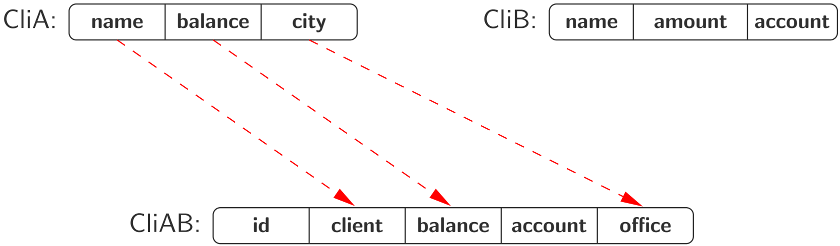

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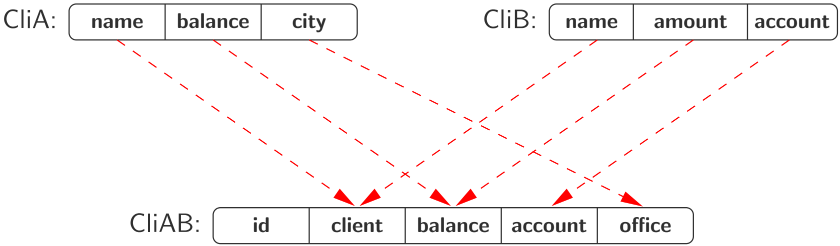

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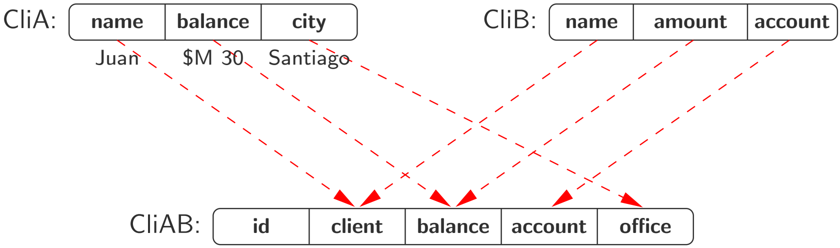

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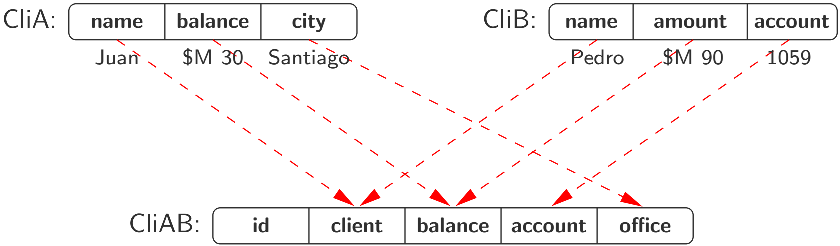

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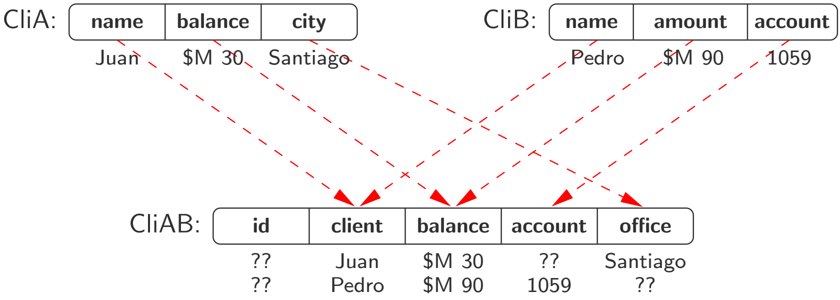

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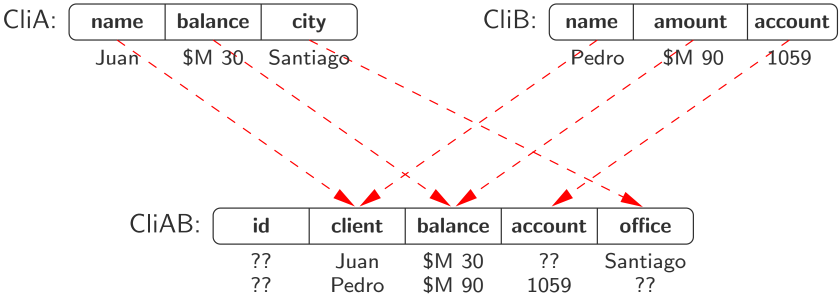

Schema mappings contain metadata

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Example:

Source: $\{\text{ClientA}(\textbf{name}, \textbf{balance}, \textbf{city})\}$

Target: $\{\text{ClientAB}(\textbf{id}, \textbf{client}, \textbf{balance}, \textbf{account}, \textbf{office})\}$

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- Formally a mapping is just a binary relation:
  the set of pairs (I, J) that satisfy the logical specification

$$\mathcal{M} = \{(I, J) \mid (I, J) \models \Sigma\}$$

In several scenarios we need to reuse the metadata in the schema mappings

Operators on schema mappings

- Composition [MH03, Melnik04, FKPT04]
- Inversion [Fagin06, FKPT07, APR08, APRR09, FN10]
- Extract, Merge [Melnik04, BM06, APRR10]

Plus some general notions for schema mappings:

- Information transfer [FKPT09, APRR10]
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We propose two new operators, *Union* and *Intersection*, using the *information transfer* notion.

Outline

Intuition and definition

Existence of union and intersection

Concluding remarks and future work

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$\mathcal{M}_2$ and $\mathcal{M}_3$ are *incomparable*

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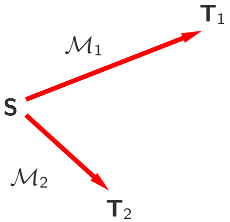

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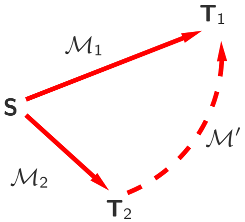

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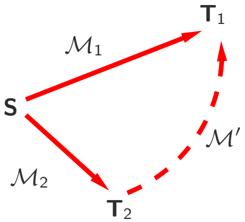

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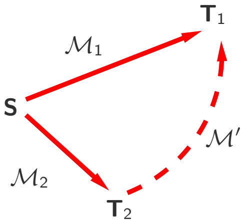

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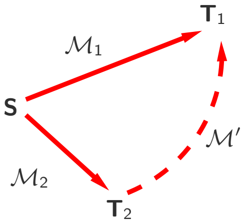

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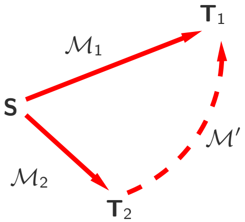

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Research questions:

- does an intersection always exist?
- what is the expressiveness needed to specify it?
- can we effectively compute an intersection?

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2. if $\mathcal{N} \in \mathcal{C}$, with $\mathcal{M}_1 \preceq \mathcal{N}$ and $\mathcal{M}_2 \preceq \mathcal{N}$, then $\mathcal{M} \preceq \mathcal{N}$

i.e. *the least upper bound*

We denote union by $\mathcal{M}_1 \sqcup_{\mathcal{C}} \mathcal{M}_2$

Intuition: the optimistic approach

Union transfers the *sum* of the info transferred by $\mathcal{M}_1$ and $\mathcal{M}_2$

for union, the same research questions apply

Outline

Intuition and definition

Existence of union and intersection

Concluding remarks and future work

$\preceq$, $\sqcap$, $\sqcup$ form a *lattice* of mappings

Theorem

There is a class of mappings $\mathcal{R}$ (that contains all the st-tgds) s.t.

$$\sqcap_{\mathcal{R}} \text{ and } \sqcup_{\mathcal{R}} \text{ always exists}$$

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$\mathcal{R}$ is the class REC of *recoverable mappings*: mappings that admit a *maximum recovery* (a notion of inverse mapping [APR08])

Another intuition based on query answering

Mapping $\mathcal{M}$, *target query* $Q_{\mathbf{T}}$, source instance I:

$$\underline{\text{certain}}_{\mathcal{M}}(Q_{\mathbf{T}}, I) = \bigcap_{(I,J) \in \mathcal{M}} Q_{\mathbf{T}}(J)$$

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for every source instance I.

- Intuitively: if $Q_{\mathbf{S}}$ is target rewritable under $\mathcal{M}$, then $\mathcal{M}$ *transfers all the source data* retrieved by $Q_{\mathbf{S}}$.

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Let $\mathcal{M}_1$ and $\mathcal{M}_2$ be mappings in $\textsc{Rec}$

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Proposition

Q is target rewritable under $\mathcal{M}_1$ and $\mathcal{M}_2$
if and only if Q is target rewritable under $\mathcal{M}_1 \sqcap_{\textsc{Rec}} \mathcal{M}_2$

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Open question

can we characterize $\sqcap_{\textsc{Rec}}$ and $\sqcup_{\textsc{Rec}}$ in terms of target rewritability?

Existence of Union

We know that $\sqcup_{\text{REC}}$ always exists, but what about st-tgds?

$$\begin{array}{llll} \mathcal{M}_2\text{:} & \mathsf{Emp}(x, y, z) & \rightarrow & \mathsf{WorksIn}(x, z) \\ \mathcal{M}_3\text{:} & \mathsf{Emp}(x, y, z) & \rightarrow & \mathsf{LivesIn}(x, y) \end{array}$$

Union:

$$\mathcal{M}\text{:} \quad \mathsf{Emp}(x, y, z) \rightarrow \mathsf{WorksIn}(x, z) \wedge \mathsf{LivesIn}(x, y)$$

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The union $\sqcup_{\textit{st-tgds}}$ always exists

That is, union of st-tgds can be specified as st-tgds

Intersection of st-tgds cannot always be specified in FO

$$\begin{array}{llll} \mathcal{M}_1: & A(x,u) \wedge B(u,y) & \rightarrow & T_1(x,y) \\ \mathcal{M}_2: & B(y,v) \wedge A(v,x) & \rightarrow & T_2(y,x) \end{array}$$

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The following mapping $\mathcal{N}_n$ is less informative than $\mathcal{M}_1$ and $\mathcal{M}_2$:

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Theorem

There is no mapping $\mathcal{M}'$ expressible in FO such that for every n

$$\mathcal{N}_n \preceq \mathcal{M}' \preceq \mathcal{M}_1 \quad \textit{and} \quad \mathcal{N}_n \preceq \mathcal{M}' \preceq \mathcal{M}_2$$

Existence of the intersection

We know that $\sqcap_{\mathrm{REC}}$ always exists but

Corollary

There are st-tgds mappings $\mathcal{M}_1$ and $\mathcal{M}_2$ such that

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Corollary

The intersection $\sqcap_{\textit{st-tgds}}$ not always exists

What is the expressiveness needed
to specify the intersection of st-tgds?

Expressing the intersection of st-tgds

Observation:

- for every $\mathcal{M} \in \textsc{Rec}$ there exists an $\mathcal{M}'$ which is the *maximum recovery* of $\mathcal{M}$ (a notion of *inverse* of $\mathcal{M}$).

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Theorem

Let $\mathcal{M}_1$ and $\mathcal{M}_2$ be mappings in $\textsc{Rec}$, $\mathcal{M}_1'$ and $\mathcal{M}_2'$ be their maximum recoveries, and let $\mathcal{N}$ be:

$$\mathcal{N} \; = \; \big[(\mathcal{M}_1 \circ \mathcal{M}_1') \cup (\mathcal{M}_2 \circ \mathcal{M}_2')\big]$$

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Then, the intersection $\mathcal{M}_1 \sqcap_{\textsc{Rec}} \mathcal{M}_2$ is the infinite composition

$$\mathcal{N} \circ \mathcal{N} \circ \mathcal{N} \circ \cdots$$

Intersection of st-tgds is expressible in SO

Theorem

Given st-tgds $\mathcal{M}_1$ and $\mathcal{M}_2$, the intersection $\mathcal{M}_1 \sqcap_{\mathrm{REC}} \mathcal{M}_2$ can be specified in Second Order logic

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Proof idea

We show that:

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We show that:

- for an st-tgd mapping $\mathcal{M}$ the composition of $\mathcal{M}$, with its maximum recovery can be specified in FO

Intersection of st-tgds is expressible in SO

Theorem

Given st-tgds $\mathcal{M}_1$ and $\mathcal{M}_2$, the intersection $\mathcal{M}_1 \sqcap_{\mathrm{REC}} \mathcal{M}_2$ can be specified in Second Order logic

Proof idea

We show that:

- for an st-tgd mapping $\mathcal{M}$ the composition of $\mathcal{M}$, with its maximum recovery can be specified in FO
- the infinite composition of mappings specified in FO, can be specified in SO

Outline

Intuition and definition

Existence of union and intersection

Concluding remarks and future work

Union and Intersection of schema mappings

We have proposed two new operators $\sqcup$ and $\sqcap$

- based on information transfer
- applications in mapping generation and query answering
- can be used to define new operators: e.g. *difference*

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- can be used to define new operators: e.g. *difference*

Future work include

- more on expressiveness of intersection:
  - do we really need SO? can we use less expressiveness?
  - when the intersection is expressible in FO or as st-tgds?
- complexity
  - is it decidable if $\mathcal{M}$ is the intersection of $\mathcal{M}_1$ and $\mathcal{M}_2$?

Union and Intersection of Schema Mappings

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