# A Practice-Oriented Treatment of Pseudorandom Number Generators

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# Talk Outline

- Background
- Security definitions
- Analysis of the two most widely-used PRNGs (ANSI X9.17 and FIPS 186)
- Other security considerations

# Background (1)

- Cryptographic PRNGs are required for virtually every cryptographic application.
- There exist provably-secure PRNGs under number-theoretic assumptions [BM84,BBS86,G00].
  - Not the most popular ones: efficiency is an issue.

# Background (2)

- Most popular PRNGs use block ciphers or hash functions as the underlying primitive
- Standardized PRNGs
  - The ANSI X9.17 PRNG
  - The FIPS 186 PRNG

There have been no security proofs (under any reasonable assumption) that these PRNGs are secure.

# Related Work

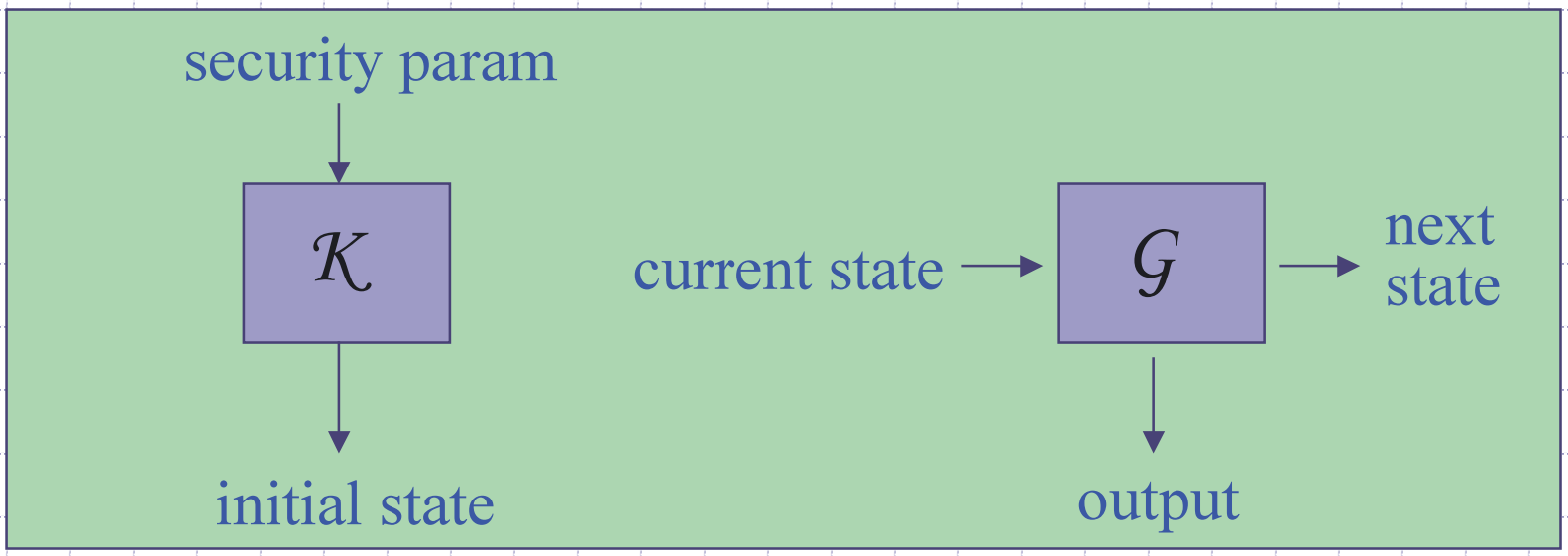
- There is extensive literature on the theory of PRNGs [Y82,BM84,BBS86,HILL89].
- Results on block-cipher-based PRNGs focus on provably-secure design [ARV99] and generic forward security techniques [BY01,AB00].
- Previous analyses [KSWH98,G98,B01] identified weaknesses but were mostly ad-hoc.

# Our Contributions

- Analysis framework more suitable for PRNGs as used in practice
- Analysis of the ANSI X9.17 and FIPS 186 PRNGs
  - Formalize assumptions on primitives
  - Suggest guidelines on secure usage
  - Identify improvements

# PRNGs in cryptography

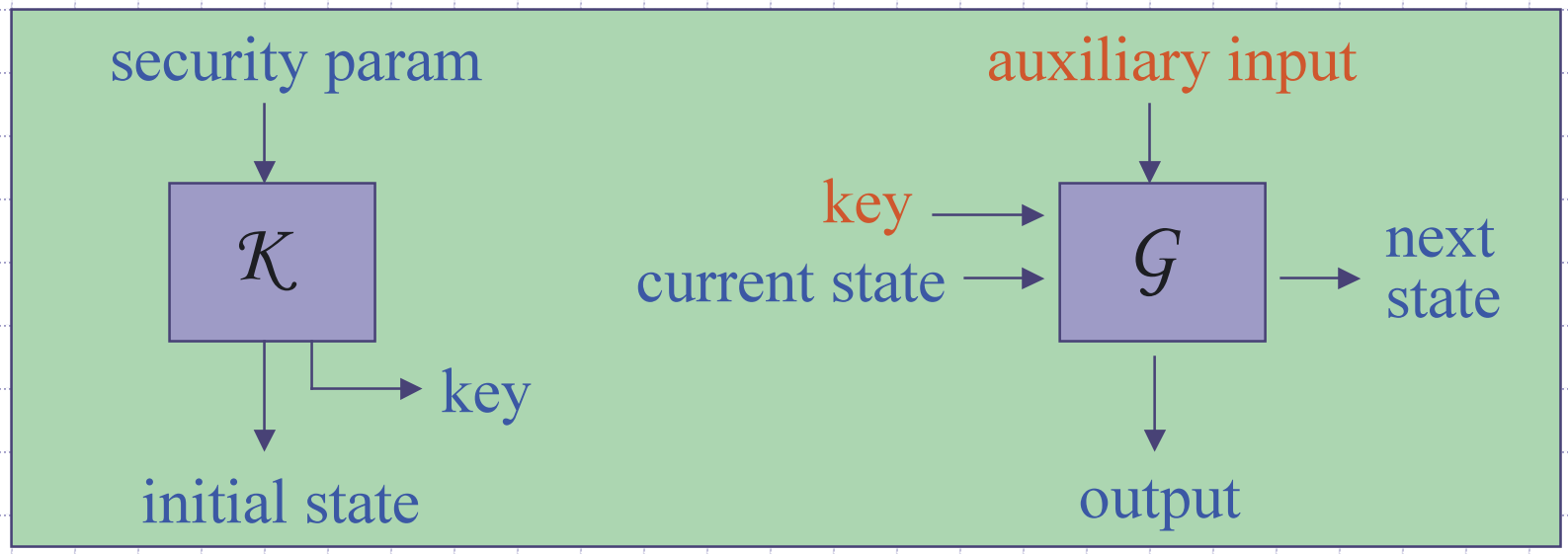
A PRNG $\mathcal{GE} = (\mathcal{K}, \mathcal{G})$ is a pair of stateful algorithms




$\mathcal{G} : current\ state \rightarrow next\ state \times output$

# PRNGs as used in practice

PRNGs are extended so $\mathcal{G}$ takes additional inputs




$$\mathcal{G} : key \times current\ state \times auxiliary\ input \rightarrow next\ state \times output$$

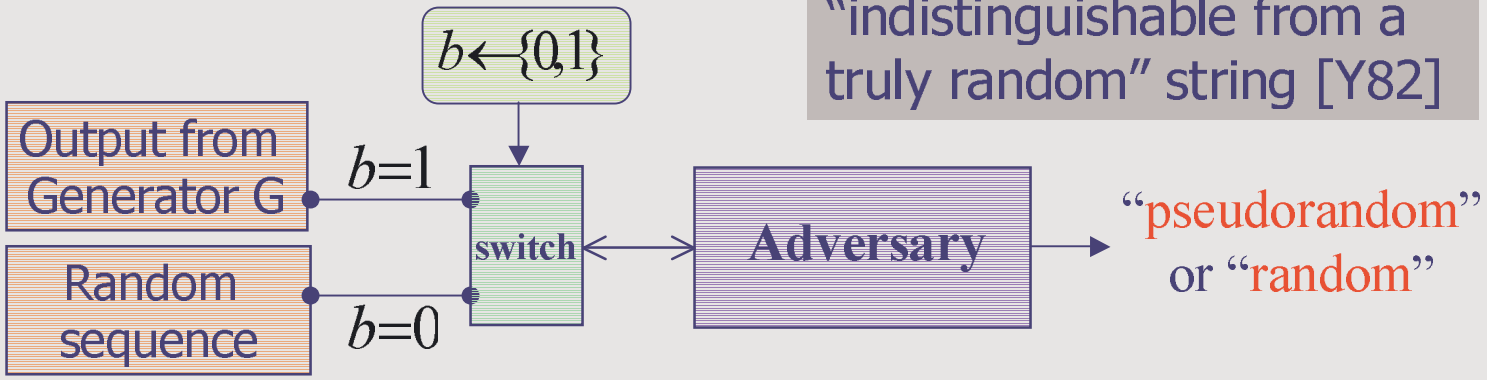
# PRNGs: Theory vs. Practice

| PRNGs used in cryptography | PRNGs used in practice |
| --- | --- |
| States are assumed hidden at all times | ◆ Take "auxiliary inputs" (e.g. timestamps)<br>◆ May leak out current state over time<br>◆ Are based on secret-key or keyless primitives |

# Towards a security definition (1)

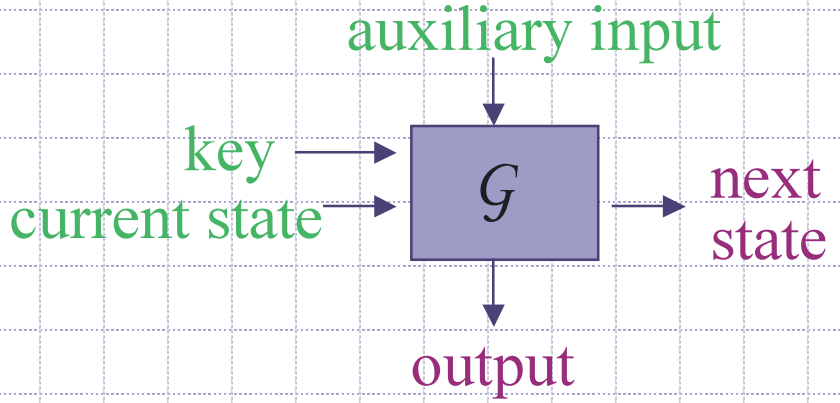
## Security goal

The output should be "indistinguishable from a truly random" string [Y82]




## Attacker Capabilities

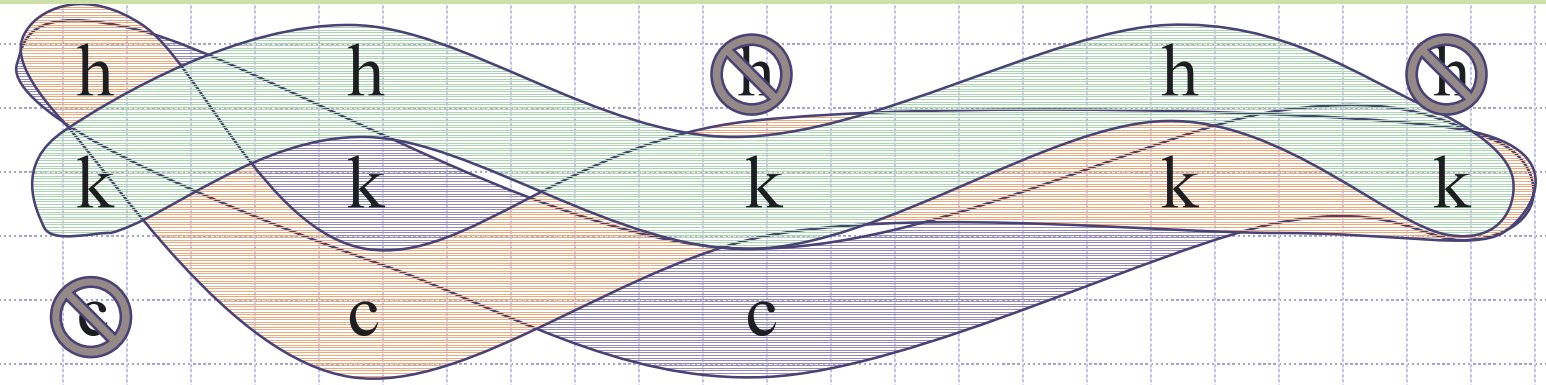
| Inputs | Outputs |
|---|---|
| hidden<br>known<br>chosen | hidden<br>known |



# Towards a security definition (2)

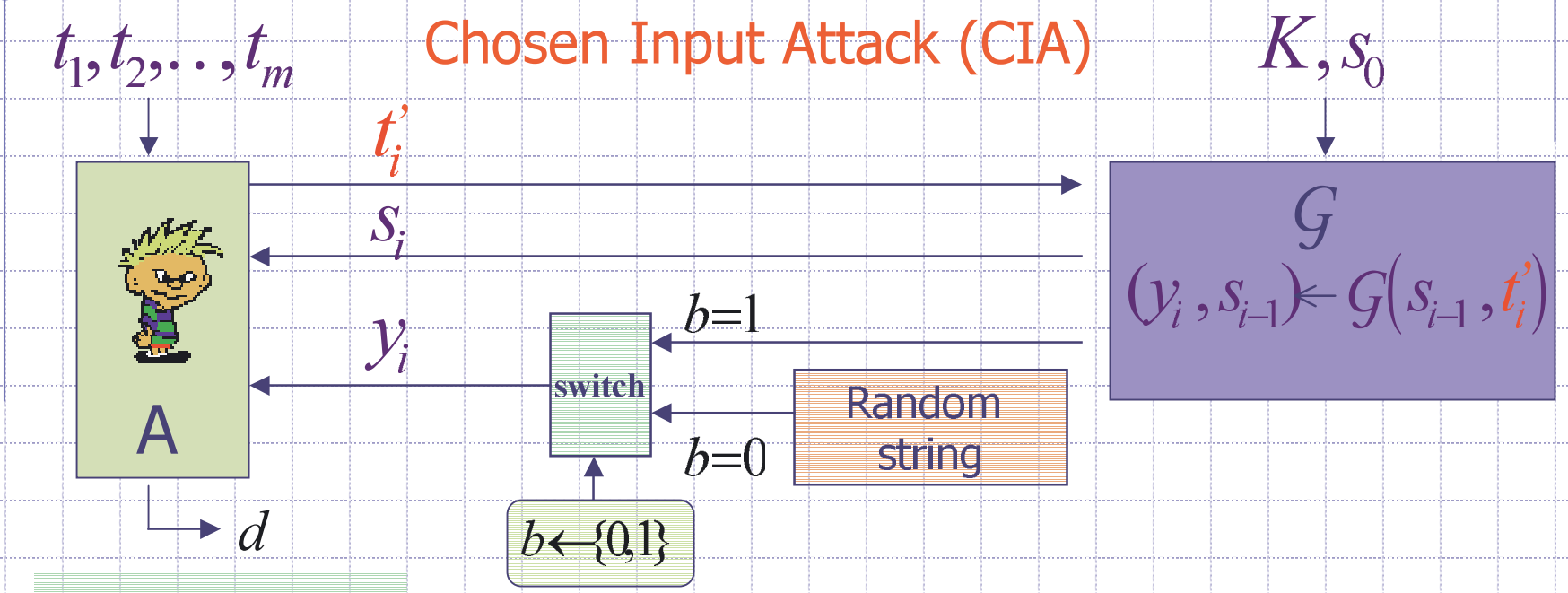
## Attacker Viewpoint

$$G : key \times current\ state \times auxiliary\ input \rightarrow next\ state \times output$$




| Attack Name | key | current state | aux input | next state |
|---|---|---|---|---|
| **Chosen Input Attack** | hidden | known | chosen | known |
| **Chosen State Attack** | hidden | chosen | known | known |
| **Known Key Attack** | known | hidden | known | hidden |

# Security of PRNGs (1)

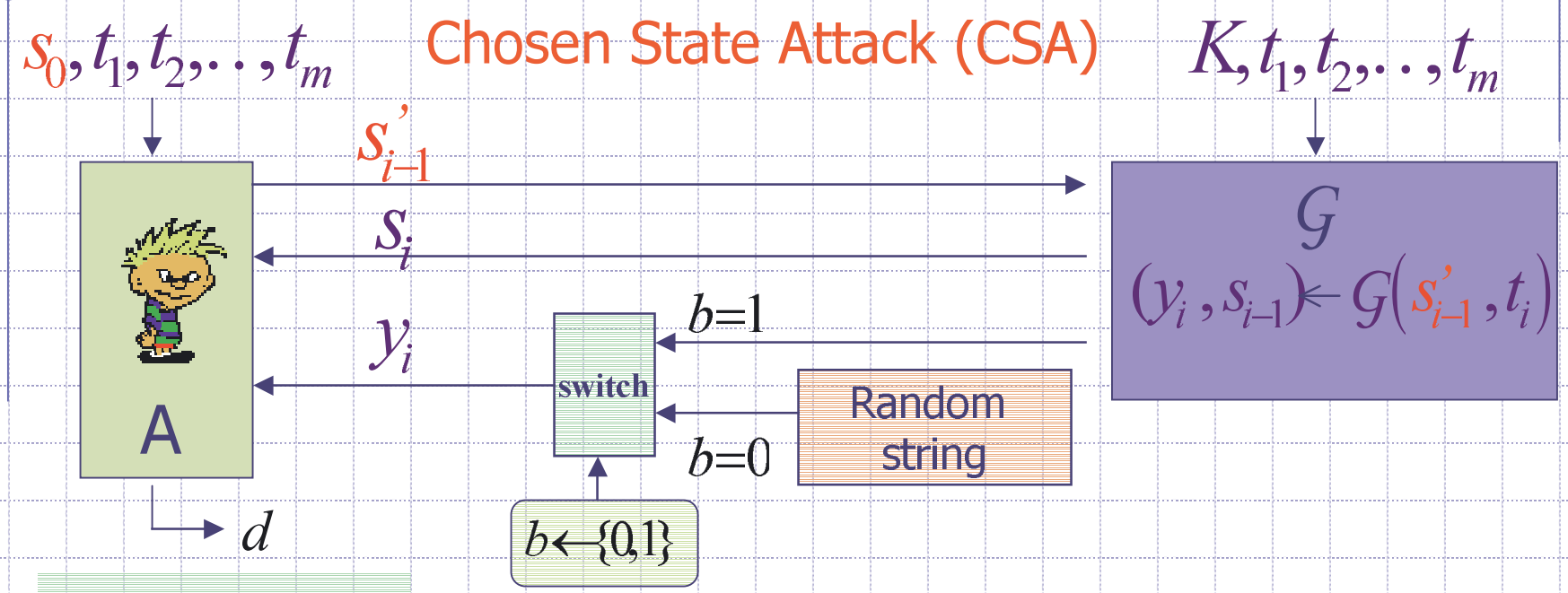



A wins if $d=b$

$$\mathrm{Adv}^{\text{prg-cia}}_{\mathcal{GE},m}(t) = \max_A \{2\Pr[A \text{ wins}] - 1\}$$

We want $\mathrm{Adv}^{\text{prg-cia}}_{\mathcal{GE},m}(t)$ "small" for "large" $t$

# Security of PRNGs (2)

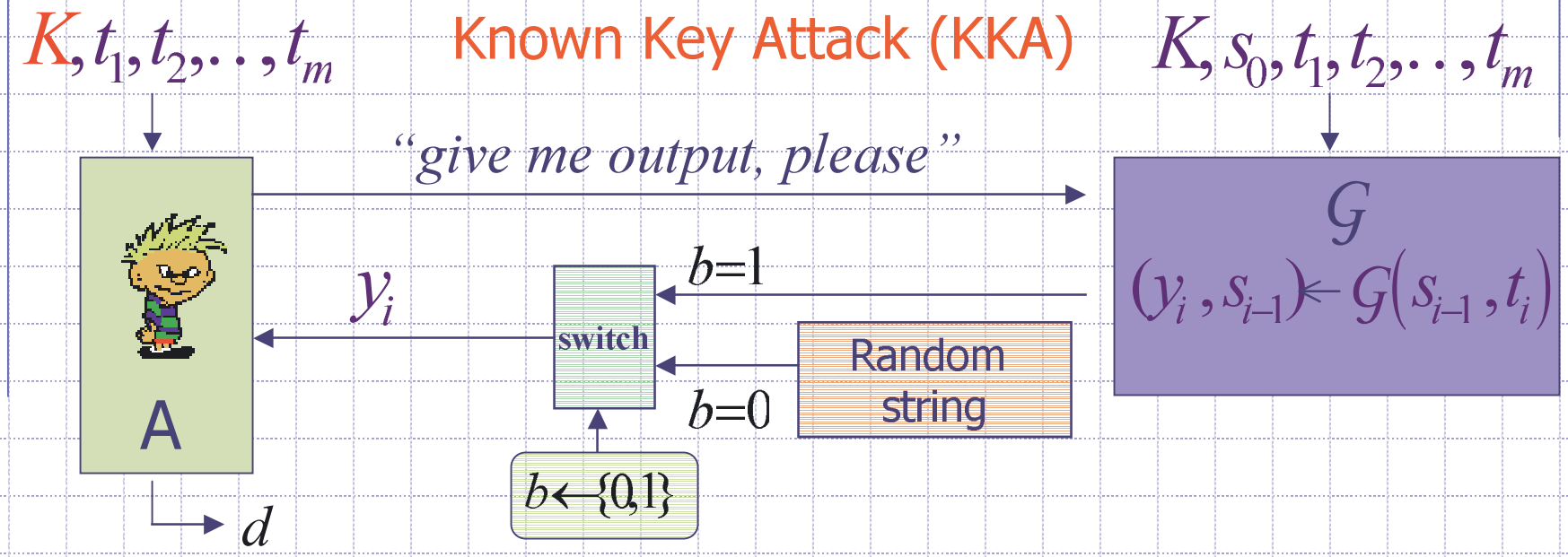



A wins if $d=b$

$$\mathrm{Adv}^{\text{prg-csa}}_{\mathcal{GE},m}(t) = \max_A \{2\Pr[A \text{ wins}] - 1\}$$

We want $\mathrm{Adv}^{\text{prg-csa}}_{\mathcal{GE},m}(t)$ "small" for "large" $t$

# Security of PRNGs (3)

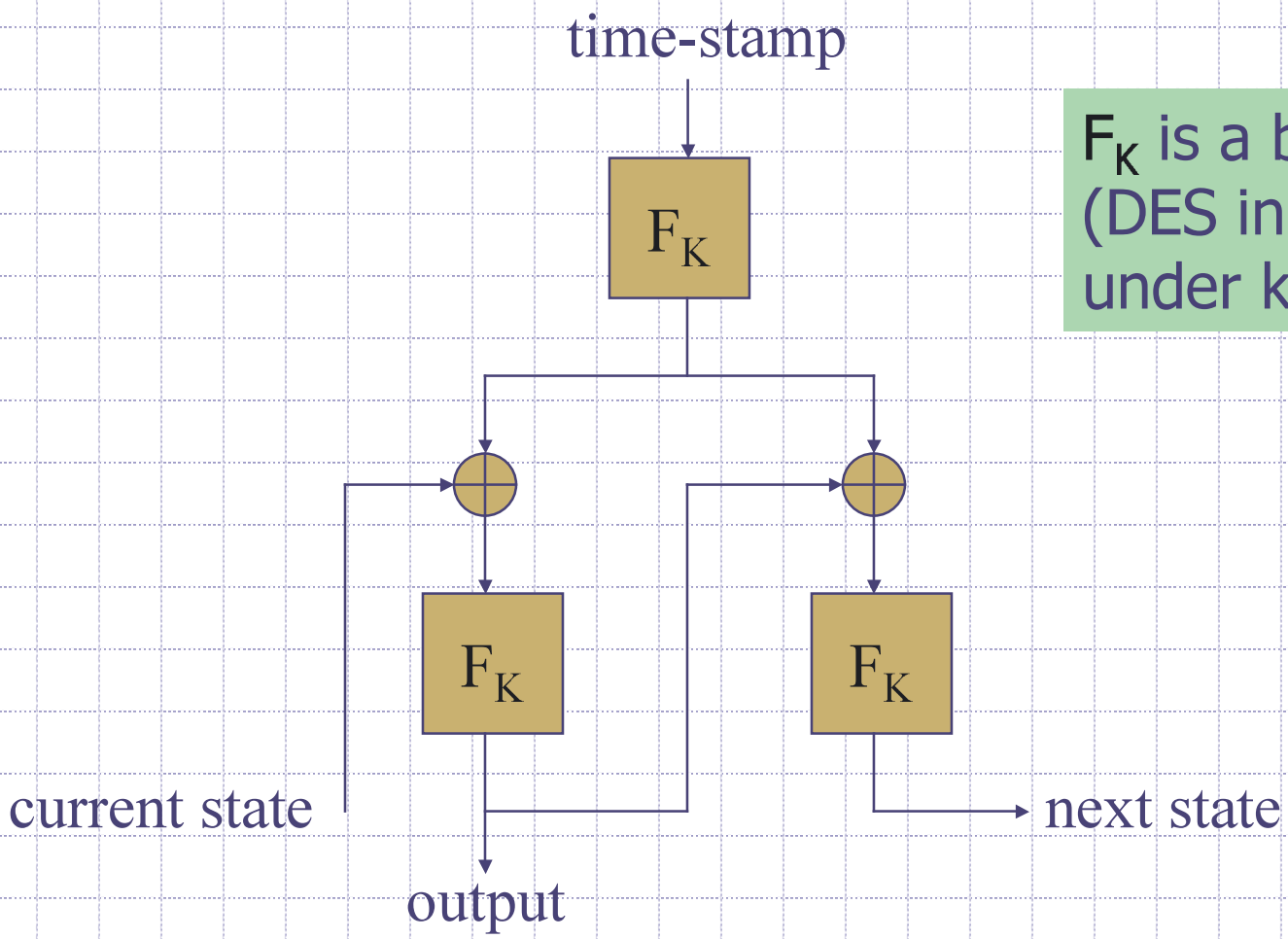



A wins if $d=b$

$$\mathrm{Adv}^{\text{prg-kka}}_{\mathcal{GE},m}(t) = \max_A \{2\Pr[A \text{ wins}] - 1\}$$

We want $\mathrm{Adv}^{\text{prg-kka}}_{\mathcal{GE},m}(t)$ "small" for "large" $t$

# The ANSI X9.17 PRNG




$F_K$ is a block cipher (DES in the spec) under key K

# ANSI PRNG: Security results (1)

- Insecure under any attack if key is known.
- Insecure under an attack where both the input and current state may be chosen.

# ANSI PRNG: Security results (2)

ANSI PRNG is secure under Chosen Input Attack and Chosen State Attack assuming the underlying block cipher is a pseudorandom permutation (PRP).

Theorem: Let $\mathcal{GE}$ be the ANSI X9.17 PRNG based on a function family $F$. Then

$$Adv^{prg-csa}_{\mathcal{GE},m}(t) \leq 2 \cdot Adv^{prp}_{F}(t,3m) + m \cdot (13m-2) \cdot 2^{-n-1}$$

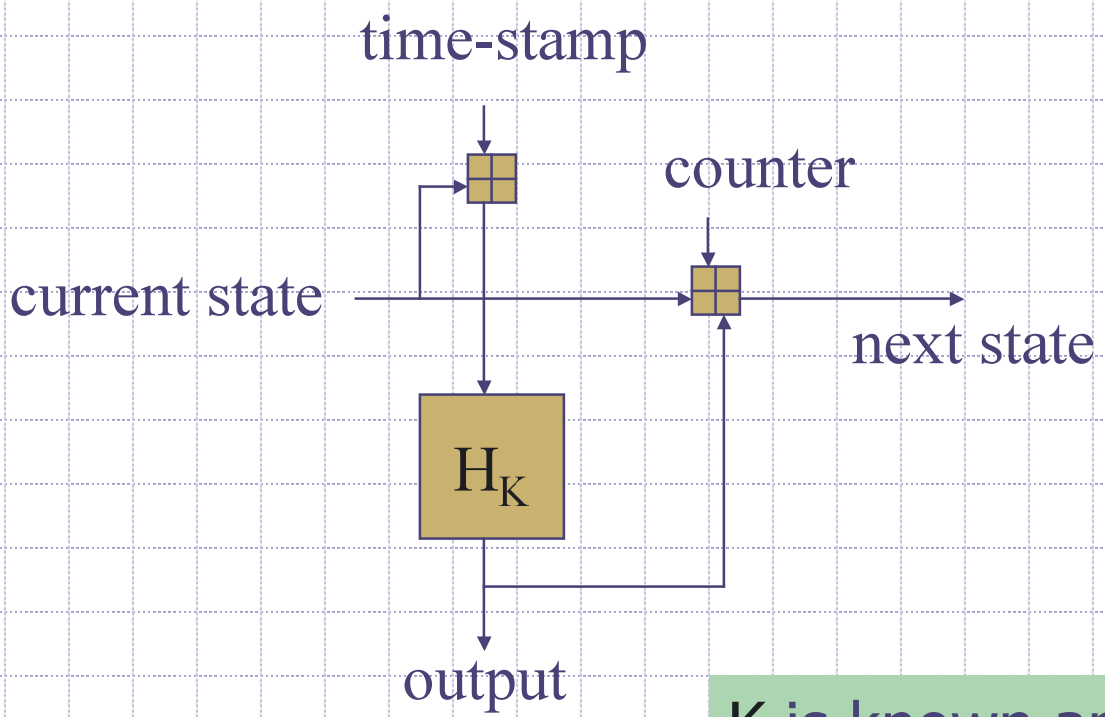
$$Adv^{prg-cia}_{\mathcal{GE},m}(t) \leq 2 \cdot Adv^{prp}_{F}(t,3m) + \left((4m-1)^2 + m^2 + 1\right) \cdot 2^{-n-1}$$

where $m$ is the number of $n$-bit output blocks.

# ANSI PRNG: Remarks

- Throughput can be doubled by outputting intermediate states as part of PRNG output
  - Secrecy of intermediate states is unnecessary
  - Intermediate states are pseudorandom
- "Good" randomness is better used on key (rather than on state)

# The FIPS 186 PRNG




K is known and fixed in FIPS 186 specification

# FIPS PRNG: Attacks

- Insecure under any attack where state is known.
- Insecure under any attack if the input may be chosen [KSWH98].

# FIPS PRNG: Towards an Analysis

We need reasonable assumptions on H

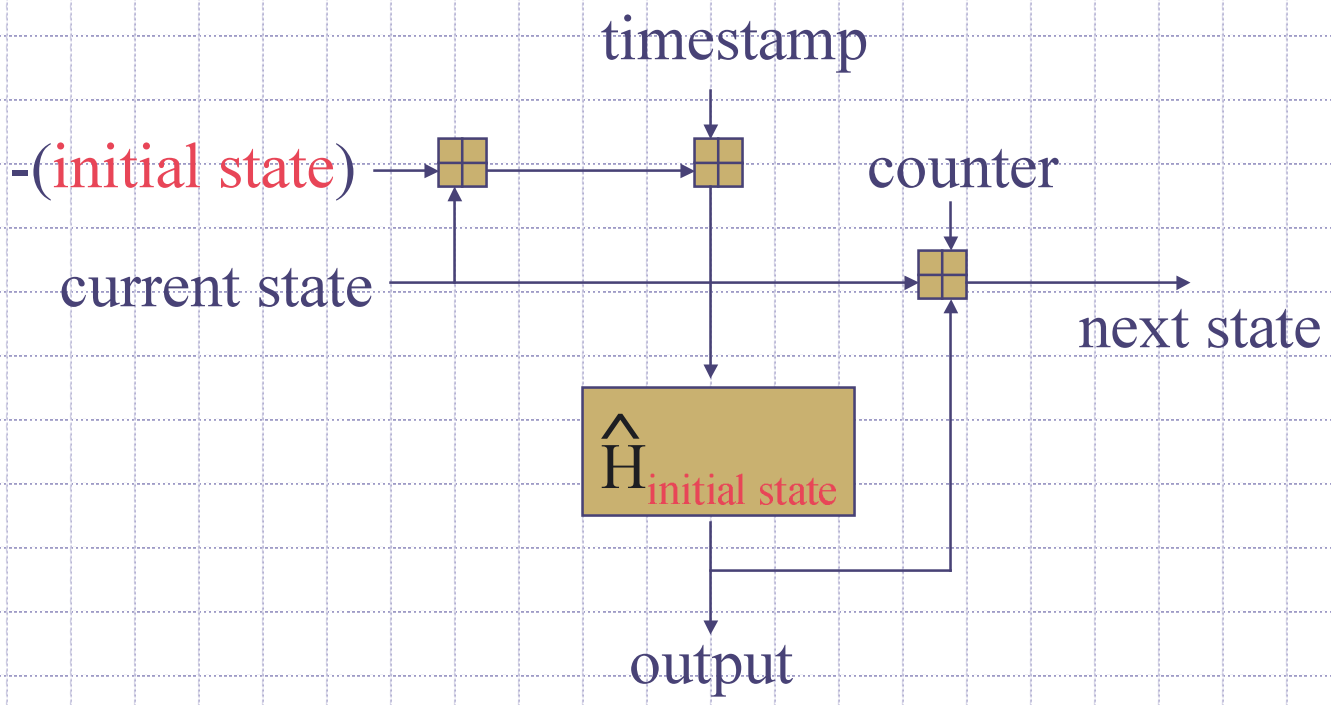
- Collision Resistance? does not suffice
- Random Oracle? overkill

$\hat{H}_s(x) = H_K(s+x)$ can be seen as secret-key hash function if s is secret

→ Assume $\hat{H}_s$ is a PRF family

- Similar assumptions have been made before [BGR95,BCK96a,BCK96b,ARV99].
- No known attacks seem to contradict this assumption.

# FIPS PRNG: An alternative view




Different looking implementation but same input/output characteristics

# FIPS PRNG: Security results

FIPS PRNG is secure under Known Key Attack assuming the underlying primitive (in the alternative view) is a PRF.

Theorem: Let $\mathcal{GE}$ be the FIPS 186 PRNG based on the function family $\hat{H}$. Then

$$Adv^{prg-kka}_{\mathcal{GE},m}(t) \leq 2 \cdot Adv^{prf}_{\hat{H}}(t,m) + m \cdot (m-1) \cdot 2^{-n-1}$$

where $m$ is the number of $n$-bit output blocks.

# Other Considerations

- Most other PRNGs used in practice bear similarities with the two PRNGs analyzed
- Preserving security even under a break-in (Forward Security) seems desirable…

  But neither the ANSI nor the FIPS PRNG are forward-secure.

# Conclusions

- We propose a framework more suitable for PRNGs as used in practice
- ANSI X9.17 PRNG
  - Secure if either state or inputs are not chosen
  - Randomness is better used in key
  - Throughput can be doubled by outputting state
- FIPS 186 PRNGs
  - Secure if states are hidden and inputs are not chosen
- For both, we formalize assumptions needed on primitives.